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Determining the minimum percentage of vehicles equipped with a uCAN necessary to accurately estimate the traffic speed

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L.S. PRIEM

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MSc thesis APPLIED MATHEMATICS

"Determining the minimum percentage of vehicles equipped with a uCAN necessary to accurately estimate the traffic speed"

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Abstract

In this thesis the use of in-vehicle data, obtained from a uCAN, for the estimation of the traffic speed is investigated. Since vehicles are not equipped with a uCAN yet, the uCAN data is emulated. For this emulation, the traffic simulation model Fosim is used. With the emulated uCAN data, the minimum percentage of vehicles equipped with a uCAN that is necessary to accurately estimate the traffic speed is determined. This minimum percentage is equal to the minimum percentage for which the corresponding estimator for the traffic speed satisfies the accuracy requirement of NDW. The obtained percentage is applied to practical settings to generalize this result from a road section to the highway network of the Netherlands. Since the estimator for the traffic speed, based on data from inductive loop detectors, that is used nowadays does not satisfy the NDW requirement, the necessary minimum percentage of vehicles with a uCAN is also determined for two weaker requirements. The results based on these requirements are also generalized to the Dutch highway network. All three obtained necessary minimum percentages of vehicles with a uCAN for the three requirements also hold in the practical settings.

Preface

This master thesis is a cooperation between Rijkswaterstaat, part of the Dutch Ministry of Infrastructure and the Environment, and Delft University of Technology. The research is performed at ITS Edulab, which is the Dutch traffic and transport laboraty where students solve traffic related issues for Rijkswaterstaat.

I would like to thank my graduation committee: Johan Dubbeldam from Delft University of Technology and especially my daily supervisors Geurt Jongbloed from Delft University of Technology and Lidwien Visser-Goossens from Rijkswaterstaat Centre for Transport and Navigation for the discussions and feedback during the project. I would also like to thank my family, friends and Sander for their support throughout the project.

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Contents

1 Introduction

The Netherlands is densely populated and has a infrastructure that is highly utilized. The Dutch highways are therefore crowded, which often leads to congestion and accidents. Because of its intensely used highway network, the Netherlands is deploying Intelligent Transport Systems (ITS) [1] to contribute towards more efficient, safer and cleaner transportation. ITS makes the best possible use of the existing road network by harnessing real-time information and communication technologies and applying this to transport. As traffic is expected to grow between now and 2020 [2], further development of ITS is needed to facilitate the high level of mobility. This development focuses on the innovations of technological applications used in road traffic and particularly in traffic management, informing and guiding traffic to reduce congestion for example. In the Netherlands traffic management of highways is assigned to Rijkswaterstaat, part of the Dutch Ministry of Infrastructure and the Environment. The technological innovations can help to improve traffic management and thus are vital for the accessibility in the Netherlands. An improvement of this accessibility is beneficial for every road user, because reaching destinations from various locations will then be easier.

To improve the accessibility, the Better Utilisation programme [3] was launched in the Netherlands. This programme is aimed at reducing congestion on specific road sections in the most congested areas and facilitating the increasing use of the infrastructure. ITS plays an important role within the Better Utilisation programme, since accurate real-time information on the situation of the road side is needed to communicate real-time information on the situation of the road network to offer more precise traffic information for traffic management. This information is used to regulate traffic by informing road users so that they can obtain a clearer view of the traffic situation. Information based on current and place specific traffic conditions allows road users to make safer, well-informed decisions and adjust their behavior before and during their journeys. An example of a technological application that makes use of the real-time information is the dynamic maximum speed, where, for example, maximum allowed speeds can be temporarily decreased to avoid or reduce congestion.

Many traffic management applications rely on the communication of real-time data to improve the utilization of the existing transport system. This data is used to control the traffic with measures that inform and, if necessary, warn road users about the traffic situation. To obtain the information for these measures, data needs to be collected from traffic data collection systems, such as inductive loop detectors, laser radar, and license plate recognition cameras. The sensors used for these systems provide traffic data about current traffic conditions. Inductive loop detectors are the most widely used traffic sensors to gather data for traffic management [4]. With the data given by inductive loop detectors, variables like the speed of a vehicle that passes the detector can be computed. For dynamic maximum speeds, average speeds of traffic on road sections are needed to dynamically adapt the maximum speed at certain points. These average speeds can also be computed with the data from detectors. After processing the data from the inductive loop detectors, the traffic situation can be analysed and measures can be considered to regulate and control the traffic.

1.1 Focus of this thesis

As mentioned above, the provided data that supports traffic management applications is mostly gathered from inductive loop detectors nowadays. This method of gathering data is an established standard for providing traffic information to reduce congestion. However, detectors only provide a limited set of variables, which can be used to a limited set of traffic information, such as the speed of a vehicle and the traffic intensity. Since innovations of traffic management applications lead to the demand of detailed information from vehicles, alternative ways to collect the necessary data are investigated. It is also investigated if this detailed vehicle information can be used for applications for other traffic related fields. For example, applications concerning weather conditions on the road would like to use information about the windscreen wipers of vehicles. New traffic data collection systems that are able to provide data about more variables can improve the information that is used for traffic management, road improvement and congestion detection for example.

In the last decade, more and more advanced technology is installed in new vehicles, which means that today's vehicles are equipped with many sensors. An obvious idea for a new traffic data collection system would be to use these sensors to extract relevant data that is already stored in vehicles. Detailed vehicle data, such as steering position, indicator switch and speed, is measured frequently by the sensors in the vehicle and only needs to be collected in order to make it available for traffic management. One way to do this is by equipping vehicles with a uCAN (universal Controller Area Network), which is a universal module placed inside a vehicle to collect data from that vehicle. With this in-vehicle data, more real-time information about congestion, traffic speeds and the condition of the road is available. To test if this data of the uCAN of vehicles can be used for different traffic management applications, some vehicles in Eindhoven are equipped with a uCAN. Results of the analysed data of these vehicles demonstrate that uCAN data already can improve some of the traffic management applications [5].

The purpose of this thesis is to contribute towards the process of using in-vehicle data for traffic management, where it is investigated when in-vehicle data can be used for the estimation of the traffic speed. The traffic speed is defined as follows:

Traffic speed
The average speed of vehicles that pass a specific point in one minute, calculated for one direction of traffic.

Computing the traffic speed is important for providing information to regulate traffic and to reduce congestion, e.g. with dynamic maximum speeds. Nowadays, the traffic speed is computed with data from inductive loop detectors. An alternative method to compute the traffic speed is to make use of in-vehicle data. This in-vehicle data is referred to as uCAN data in this thesis, since in this thesis the method to collect data of vehicles is by equipping vehicles with a uCAN. With the speeds of vehicles with a uCAN, the traffic speed can be estimated. However, since the uCAN is a new module, vehicles are not yet equipped with a uCAN (except for the vehicles used in the test in Eindhoven), so traffic speeds cannot be estimated with uCAN data at this time. Vehicles first need to be equipped with a uCAN before the uCAN can be used as a method to gather vehicle speed data for measures of traffic speeds. However, not all vehicles can be equipped with a uCAN at the same time. During the installation of the uCAN into vehicles, the percentage of vehicles with a uCAN increases. These vehicles can already provide their speeds to the traffic data collection system. The traffic speed then can be estimated with only these vehicle speeds, not taking into account the speeds of vehicles without a uCAN. The question that arises is whether this traffic speed, obtained by a certain percentage of vehicles with a uCAN, is accurate enough. Moreover, if it is possible to give an accurate estimate of the traffic speed when only a certain percentage of the vehicles is equipped with a uCAN, uCAN data can be used in an earlier stage to estimate traffic speed for traffic management applications.

This leads to the main question and focus of this thesis:

What is the minimum percentage of vehicles equipped with a uCAN that is necessary to accurately estimate the traffic speed?

As a result, if this question is answered with a certain percentage and if, in the future, that percentage of vehicles equipped with a uCAN is reached, then the traffic speed can be accurately estimated with uCAN data and used for traffic management. This could mean that in the future inductive loop detectors are no longer necessary to obtain vehicle speeds for the purpose of determining traffic speeds. When the other functions of the detectors also can be substituted by uCAN data for example, the detectors no longer need to be maintained or even can be removed.

1.2 Thesis outline

In Chapter 2, the background of traffic management will be discussed. In this section, detailed information about inductive loop detectors can be found. To process the data of the detectors and make it available for traffic management, the traffic signaling system is used, which will also be described in this chapter. Last in this chapter, the uCAN will be discussed, since in this thesis uCAN data is used for the estimation of the traffic speed. However, uCAN data is not available yet and therefore needs to be emulated. The simulation model Fosim is used to simulate data that can be used to emulate uCAN data. In Chapter 3 this model will be described together with the choice for the model and the simulation settings to obtain data for the emulation of uCAN data. The obtained data in this chapter will be used in Chapter 4 for the emulation of uCAN data. Since the emulated uCAN data is used to estimate the traffic speed, Chapter 5 will discuss the estimator of the traffic speed and its distribution. In Chapter 6 this estimator is used to determine the necessary minimum percentage of vehicles based on different assumptions. The results in this chapter will be generalized in Chapter 7. After this chapter, the answer to the main question of this thesis will be given in Chapter 8. At the end of this thesis, in Chapter 9, different recommendations for future research will be discussed.

2 Background of traffic management

Every day hundreds of thousands of vehicles make use of the Dutch highways to reach their destination. For example, in 2011 one of the busiest highways was the A4, with 238, 100 vehicles on average per working day [6]. Every highway consist of at least two lanes, the A4 even is at some road sections a five-lane highway. The number of lanes of a highway determines the capacity of that highway. The capacity is defined by the maximum number of vehicles per hour at the highway with a specified number of lanes. For example, a two-lane highway, with or without an entrance ramp, has a capacity of 4200 vehicles per hour. More information and specific values of the capacity can be found in [7]. The vehicles that drive on the highway interact with each other and influence the overall movement of traffic. To improve the traffic flow by making the utilization of highways more efficient, the traffic needs to be controlled and managed. Traffic management is used to reduce congestion and improve road safety. It involves the use of applications such as travel time prediction and extra lanes at rush hour. These traffic management applications need real-time information about the situation of the traffic, which is mostly obtained from inductive loop detectors nowaday. In Section 2.1, these inductive loop detectors are discussed. The data from detectors is collected by a comprehensive traffic signaling system. This system, which is described in Section 2.2, computes variables like the speed of a vehicle. The data that is computed by the traffic signaling system is sent to the National Data Warehouse for Traffic Information (NDW) [8], described in Section 2.3. After it is checked by NDW that the data meets certain standards, measures based on data obtained from the detectors can be used to improve the traffic flow. As mentioned in the previous chapter, technological innovations can contribute towards a better utilizations of the capacity of highways. For these innovations, more detailed vehicle data is needed. Since in this thesis the uCAN is used to obtain real-time data from vehicles for the improvement of traffic management, this alternative traffic data collection system is described in Section 2.4.

2.1 Inductive loop detectors

Since the early 1960s, inductive loop detectors have been in use for the detection of vehicles and have nowadays become the most utilized traffic sensors. An inductive loop detector is an electromagnetic detection system which consists of a wire loop embedded in the roadway pavement. Figure 1 shows an detector embedded in the highway.

Figure 1: Inductive loop detector embedded in highway

Figure 1 shows two inductive loop detectors, a system also known as double inductive loop detectors. In this thesis, the double inductive loop detector is referred to as inductive loop detector. With this detector, data of the vehicle can be obtained. The wire loops are connected to an electronics unit housed in the controller cabinet located on the side of the road, which is shown in Figure 2. When a quantity of metal, such as in a car, passes over the wire loop, the inductance of the loop is decreased and causes the electronics unit to send a pulse to the controller, indicating the passage of a car, a truck or another vehicle.

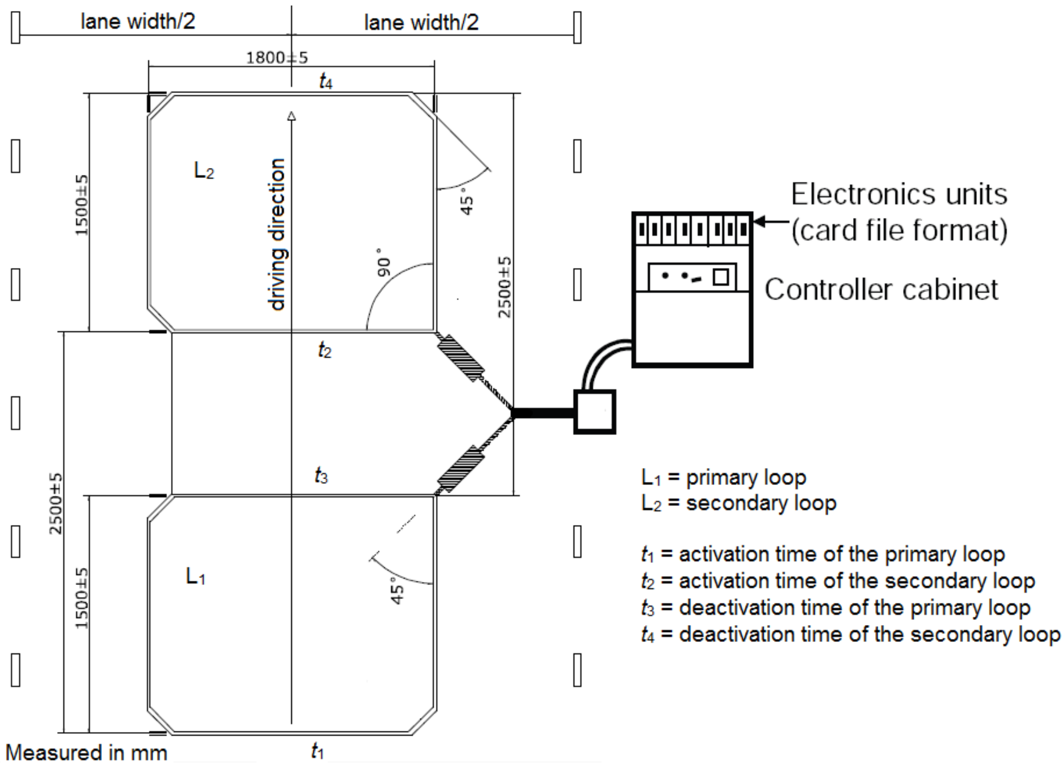


Figure 2: Inductive loop detector

As can be seen in Figure 2, the inductive loop detector is placed in the middle of the lane width. Furthermore, one loop has a length of approximately 1.5 metres, and the total length of inductive loop detector that is required to obtain data of vehicles is four meters. The vehicle first passes the primary loop L_1, where the activation time t_1 is defined as the time that the quantity of metal of the vehicle decreases the inductance of the loop. At t_3, the inductance is increased since the vehicle then has passed the primary loop. Second, the vehicle passes the secondary loop L_2, which lead to an decreasing and increasing inductance of this loop at t_2 and t_4 respectively. With these times, the driving time and the coverage time of a vehicle can be computed. The definitions of these two terms will be given in the next section. With this raw data, the length of each vehicle and the speed and intensity at the location where the detector is placed can be computed by the traffic signaling system, which will also be described in the next section.

Nowadays, inductive loop detectors form the basis of the data used to manage the traffic speed. However, there are disadvantages to the use of inductive loop detectors. One of these disadvantages is that they are localized, so they can only provide information about fixed locations to road operators and road users. Therefore, the available data may be inadequate to manage traffic in situations like incidents. Other disadvantages are their installation, maintenance and repair costs, which are relatively high [9]. Installation and maintenance of detectors require lane closure and thus lead to extra costs and disruption of traffic. Furthermore, detectors are vulnerable, very sensitive to the installation process and subject to stresses of traffic and temperature, which can lead to malfunctions of the detectors.

2.2 The traffic signaling system MTM

The Dutch highway network is equipped with an advanced traffic signaling system, called the Motorway Traffic Management system(MTM system), which collects data from traffic data collection systems for the computation of data that can be used for traffic management. This system consists of three subsystems: a detector station, a roadside station and a traffic center system, which are shown in Figure 3 and will be explained below.

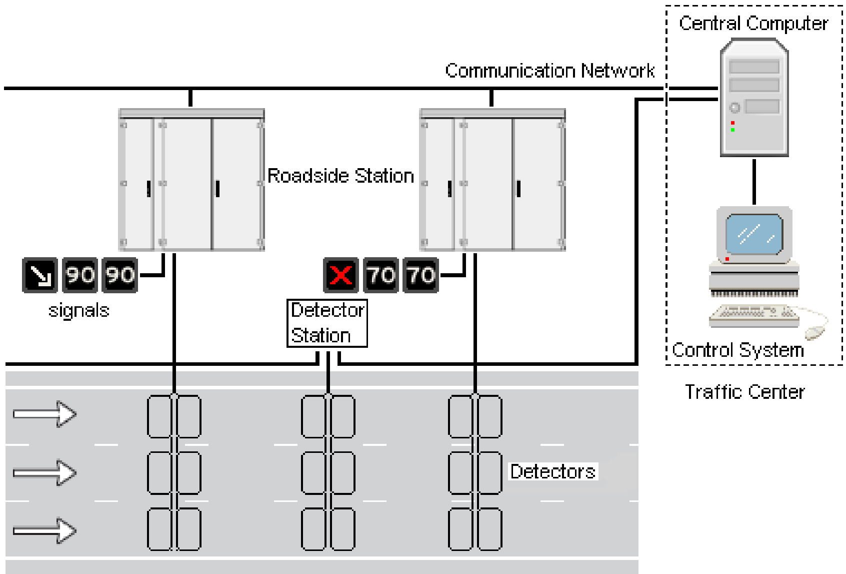


Figure 3: MTM systems

The detector station is located along the side of the highway in the controller cabinet. It measures the varying inductance, resulting from vehicles passing, of the inductive loop detectors shown in Figure 2 for example. On this basis, the detector station computes the driving time and the coverage time [10]. The driving time of a vehicle is defined as the time needed for a vehicle to pass 2.5 meters, which is the distance of the beginning of the first loop to the beginning of the second loop as can be seen in Figure 2 The coverage time of a vehicle is defined as the time that a loop has been occupied, so it is the difference between the time in which one loop has passed from unoccupied to occupied and the time in which the same loop has passed from occupied to unoccupied. Both variables are given in milliseconds.

The driving time can be computed based on the times described in Section 2.1. When a vehicle passes the inductive loop detector, first the difference between the activation times t_1 and t_2 and the difference between the deactivation times t_3 and t_4 are computed. Then it is determined whether the first or the second driving time, $t_2 - t_1$ or $t_4 - t_3$ respectively, can be considered as the most representative driving time. This last determination is carried out according to the following formula, based on [11]:

$$\text{driving time} = \begin{cases} \max\{t_2 - t_1, t_4 - t_3\} & \text{if } |(t_2 - t_1) - (t_4 - t_3)| > 0.125 \cdot \max\{t_2 - t_1, t_4 - t_3\}, \\ t_2 - t_1 & \text{if } |(t_2 - t_1) - (t_4 - t_3)| \leq 0.125 \cdot \max\{t_2 - t_1, t_4 - t_3\}. \end{cases}$$

So when the absolute value of the difference of both driving times is bigger than 12.5% of the largest value of the two driving times, then the largest value of the two driving times is chosen as the most representative driving time. When the difference is smaller than or equal to 12.5% of the largest value of the two driving times, the driving time $t_2 - t_1$ is chosen as the most representative driving time.

To compute the coverage time that can be considered as most representative, the coverage times of both the primary loop and the secondary loop are calculated. These are $t_3 - t_1$ and $t_4 - t_2$ respectively. The most representative coverage time is then defined as the maximum of both calculated coverage times, i.e.

$$\text{coverage time} = \max\{t_3 - t_1, t_4 - t_2\}.$$

After the raw data has been computed in the detector stations, the driving time and the coverage time of a vehicle are sent to the roadside station, which is also located along the side of the highway. The roadside station collects the two data points of each car and with this information it computes the length of each vehicle and the speed and intensity of the location where the double inductive loop is placed [12]. The length l in metres and the speed v in m/s of a vehicle are calculated with the following formulas:

$$l = 2.5 \cdot \frac{\text{coverage time}}{\text{driving time}} - 1.5, \qquad v = \frac{2500}{\text{driving time}}.$$

The traffic speed is then defined as the average of the speeds of the vehicles that pass the inductive loop detector. Together with the intensity and the length of the vehicles, the traffic speed is sent to the traffic center. There are five traffic centers in the Netherlands. In these centers, the data from the road station is used for traffic management. For example, when congestion is detected by decreasing traffic speeds, the traffic center sends a signal to the road station, which controls variable message signs that then show a speed limit for the vehicles. Variable message signs, as shown in Figure 4 for example, are mostly placed above the highway to inform road users about the situation on the highway.

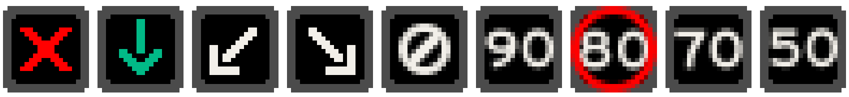


Figure 4: Variable message signs

2.3 National Data Warehouse for Traffic Information

To use the traffic speed that is calculated as described in the previous section for traffic management, the data from the MTM system is sent to NDW. Moreover, NDW collects all data concerning the traffic of the highways in the Netherlands. Every minute, data from more than 20,000 measuring sites in the Netherlands is collected by the database. With this data, real-time traffic information is obtained that can be used for traffic management. At NDW, all the data is processed, stored and subsequently redistributed to users of the data who then inform road users of the current traffic situation. One recipient of the data is Rijkswaterstaat, which uses this data to reduce congestion for example. To ensure the quality of all delivered data, NDW has requirements for the incoming data. So when other data collection systems are used, the data from these systems need to satisfy the requirements of NDW as well. In the next section, the data collection system that will be used in this thesis will be discussed.

2.4 uCAN

The technologies used in vehicles depend on sensors that are installed in the vehicle. Each sensor performs measurements, such as the speed of the vehicle or the oil level, and provides the obtained information through wires to other sensors and the technologies that need the specific information. For example, the engine needs to tell the transmission what the engine speed is. In the 1980's, the constantly growing number of sensors in vehicles, due to the innovations of technological tools, led to a growing number of necessary connections between these sensors. The complexity of wiring each sensor to every other sensor led to the creation of a central communications network in the vehicle, which was developed by Robert Bosch Gmbh and is known as the Controller Area Network (CAN bus) [13]. Sensors can be installed on this network in order to be able to communicate with any other module that is installed on the network without using redundant wiring. Since a vehicle has many sensors, several CAN busses are installed in the vehicle. The sensors that are connected to a specific CAN bus can also communicate with sensors that are connected to other CAN busses as necessary. Figure 5 shows a vehicle with its CAN busses.

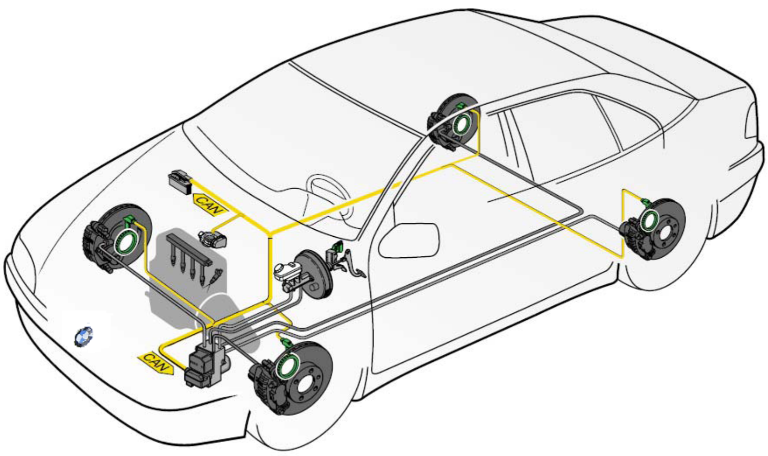


Figure 5: CAN bus in vehicle

The vehicle data from the sensors is stored in the CAN bus inside that vehicle. This means that vehicles generate a vast amount of data, such as speed, usage of breaks and usage of windscreen wipers. Compared to the data obtained from inductive loop detectors, the data from the CAN bus consists of a large variety of quantities. The CAN bus thus contains a tremendous amount of information of that vehicle. Since many traffic management applications nowadays require detailed information from the vehicle, data of the vehicle stored in the CAN bus can contribute to a more efficient and safer transportation.

However, the data that is stored in the CAN bus of a vehicle is not openly accessible. This data needs to be extracted so that it can be analysed and used for traffic management. In January 2012 the Smart in-car project was launched in the Netherlands to develop a new traffic data collection system that gathers the data from the CAN bus. By equipping a vehicle with a uCAN, developed by Beijer Automotive, a universal module that can access the data of the CAN bus, the data from the CAN bus of that vehicle can be retrieved, where also the position of the vehicle is sent via the GPS unit of the uCAN. This module is connected to a server on which all data from that vehicle is collected, so that the detailed vehicle sensor data is made available for traffic information. If a vehicle is equipped with a uCAN, the uCAN sends the data in time steps to the server. There are two types of variables in the data, the event based variables and the periodic variables. Event based variables, such as the left and right direction indicator switch, are only stored in the CAN bus when the variable is switched on or off. The uCAN thus sends this information only when the switch is changed, so the time step for this type of variables is not constant. The periodic variables, such as the speed and the location of the vehicle, are stored in the CAN bus every time step. For the Smart in-car project, the time step is set equal to one second, so the uCAN sends these variables every second to the server.

The Smart in-car project aims to improve the traffic flow and increase traffic safety by using the data from the uCAN of a vehicle. To test if the data that is provided by the uCAN can be used for applications related to road maintenance, traffic mangement and weather for example, 300 vehicles in Eindhoven are equipped with a uCAN [14]. For this test, taxi's and vehicles of the ANWB (the Dutch road users and tourist organisation) are used [15]. The reason for the contribution of the taxi's and the vehicles of the ANWB is that, with the help of cooperative systems, drivers can be individually informed about their driving behavior. Furthermore, the data can show whether the drivers follow the rules, which is of interest for the taxi company, or whether a car has a failure, which is of interest for the ANWB. In the event of a collision or other incidents, it also can provide useful information for insurers. An example uCAN data set with detailed information about the obtained data from the CAN bus can be found in [16].

The data of the 300 vehicles with a uCAN is applied in several use cases to test if uCAN data improves the above mentioned applications. LaQuSo (Laboratory for Quality Software), a partner of the Smart in-car project, has investigated four use cases in [17], concerning pothole and road roughness detection, traffic jam, driver behavior and rain detection. In all use cases, useful results were obtained, which implies that uCAN data can be used to improve traffic management and weather applications for example. In this thesis, the interest lies in the application of data from vehicles equipped with a uCAN for the estimation of the traffic speed. The uCAN data that is relevant for this estimation are the positions and the speeds of a vehicle at the corresponding times. As mentioned above, the position of a vehicle is obtained via the GPS component of the uCAN, while the speed of the vehicle is measured by the rotation of the wheels. The position and speed are therefore independently measured. Note that the positions and speeds of vehicles can also be obtained from floating car data (e.g. Nokia) or navigation systems (e.g. TomTom, Garmin) for example. As a consequence, the results obtained in this thesis for the data collected from a uCAN can be applied to any other in-vehicle data collection system from which every second data is gathered.

3 The simulation model Fosim

To estimate the traffic speed based on the speeds of vehicles equipped with a uCAN, uCAN data is needed. As mentioned above, only approximately 300 vehicles in Eindhoven are equipped with a uCAN nowadays. This means that the percentage of vehicles that are equipped with a uCAN is very small. The uCAN data from these vehicles with a uCAN contains information of the vehicles that are driving on the roads in the city centre of Eindhoven and on the A2 highway. Only the data of the latter is important in this thesis, because the traffic speed on the roads of city centres is not considered. Since approximately 70% of the vehicles with a uCAN are taxis, which are mainly present in the city centre, the percentage of uCAN data that can be used for the estimation of the traffic speed is even smaller when only the vehicles that are driving on the highway are considered. More specific, an example uCAN data set of 60 vehicles in Eindhoven, delivered by the Smart in-car project, shows that between April 2012 and October 2012 per day approximately five vehicles were driving on the highway. Moreover, these five vehicles were not at the same time present on the highway, so at a certain point the speed of only one vehicle is known for only five different minutes per day. The very low percentage of vehicles that are equipped with a uCAN nowadays makes it therefore not possible to use the data of these vehicles for the estimation of the traffic speed.

However, to give an answer to the main question of this thesis, uCAN data is needed. Since there is almost no uCAN data available yet, this data needs to be emulated. This emulated uCAN data can then be used to estimate the traffic speed. The data that is collected from the uCAN contains information about much more quantities than that is needed for the estimation of the traffic speed based on the uCAN data. Therefore, not all uCAN data has to be emulated. As mentioned above, the uCAN data of a vehicle that is relevant in this thesis are the positions and the speeds at the corresponding times. Since these positions and speeds are gathered from GPS and the CAN bus respectively, the obtained data contains measurement errors. However, emulated instead of real uCAN data is used in this thesis, so the errors of the GPS and uCAN do not need to be taken into account. The simulation model Fosim (Freeway Operations SIMulation) [18] is used in this thesis to provide the data that is needed for the emulation of relevant uCAN data. In Section 3.1 the choice for this particular traffic simulation model will be discussed. In Fosim, different models are used to simulate the behavior of the vehicles. These models will be described in Section 3.2. Section 3.3 discusses the simulation settings of the Fosim model with all parameters that are chosen in order to obtain Fosim data that will be used to emulate relevant uCAN data. The last section, Section 3.4, shows the obtained Fosim data.

3.1 The choice for the simulation model Fosim

As mentioned above, since uCAN data is not available yet for computations of the traffic speed, this data has to be emulated. In order to do this, data of the daily road traffic needs to be simulated, which can be done with macroscopic and microscopic simulation models. Macroscopic models are used to analyse the relationships among traffic characteristics, such as the density and the traffic flow. In contrast, microscopic models simulate the individual behavior of a vehicle by defining dynamic variables for the position and the speed of a single vehicle. In this type of models, detailed data about each vehicle can be obtained from these simulated individual behaviors. Since this data includes the positions and the speeds of each vehicle separately at given times, this data can be used to emulate the relevant uCAN data. Therefore, a microscopic model is chosen to simulate the traffic.

More specifically, the microscopic model that is chosen in this thesis for the traffic simulation is Fosim. Several other microscopic simulation models were considered, like Vissim (Verkehr In Städten — SIMulationsmodell) [19] and Corsim (CORridor SIMulation) [20] for example. In Vissim, the data of a vehicle can be obtained once per second, so the relevant uCAN data is emulated immediately from Vissim. However, Vissim is developed to simulate traffic for worldwide usage, and thus is not specifically developed to study traffic on Dutch highways. Neither is Corsim, because this simulation model is specified for highways in the USA. Since the properties of highways depend on different factors, like the maximum speed or the flatness of the landscape, the behavior of vehicles differ amongst countries. This research focuses on data from Dutch highways, so it is important that this behavior is modeled for the highways of the Netherlands. In contrast to both Vissim and Corsim, the simulation model Fosim is calibrated and validated for the Dutch highways ([21],[22]). For this reason, Fosim is used as simulation model for the emulation of uCAN data in this thesis. In contrast to Vissim, the data in Fosim is not given once per second and thus cannot be used as emulated uCAN data immediately. Therefore, Fosim data will be processed in Chapter 4 so that relevant uCAN data can be emulated. In the next section, the main principles of the models used in Fosim will be described.

3.2 The models used in Fosim

Fosim can be used to simulate the traffic flow process on the level of individual vehicles. A random vehicle generator is used in Fosim to place the vehicles on the lanes of the road section. At the start of the simulation, the first vehicle is placed at the origin of the road section by this generator. Based on the intensity that is given (see Section 3.3.3), the generator determines for each lane the time that a new vehicle needs to be placed. This time depends on the time since the last vehicle was placed on the lane and the desired following distance of the vehicle that will be placed. The latter is determined by the individual behavior of the vehicle specified in Fosim. This individual behavior of vehicles is based on different models. The two most important models, namely the lane change model, which is a model for lateral movements, and the car-following model, which is a model for longitudinal movements, are described in Section 3.2.1 and Section 3.2.2 respectively.

3.2.1 Lane change model

First of all, each driver of a vehicle has a desired speed which he will always attempt to reach. This desired speed can differ from driver to driver. For example, the desired speed of a truck driver will in general be lower than that of a vehicle, since the maximum speed for trucks is lower than that for vehicles. A driver can maintain his desired speed without changing lanes when that driver is uninfluenced by other traffic. However, when a driver approaches another vehicle in the same lane and cannot maintain his desired speed, he wants to change lanes to overtake that vehicle. To decide if it is possible for the driver to change lanes, three aspects are considered. First, the acceleration of the vehicle that is driving in front of the driver is considered. If this acceleration is too high, the driver decides not to change lanes, since overtaking would then take too much time. Second, the vehicle on the adjacent lane that will be driving in front of the driver if he changes lanes is considered. When this vehicle is reducing its speed at the moment of lane changing, the driver does not overtake the vehicle in front of him as well. Third, there could be a vehicle on the adjacent lane behind the driver. In that case, the driver considers the decelerations necessary for a lane change, both regarding the driver himself and of the vehicle behind him. A too high deceleration for either of them will lead to the driver's decision to not change lanes.

Only if all the above-mentioned aspects are not too high, the driver decides to overtake the slower vehicle. If the driver does not decide to overtake the vehicle in front of him, the car-following model described in the next section will be applied.

3.2.2 Car-following model

In the case that the driver approaches another vehicle and does not overtake this vehicle, he adjusts his speed to the vehicle in front of him. The desired following distance, the distance at which a driver wants to follow the vehicle in front of him, is important in this situation. This distance depends on the speed of the driver: for higher speeds the desired following distance is bigger than for lower speeds. The choice of the acceleration of the driver to attain his desired following distance is based on the distance to the car in front of him.

When the driver approaches the slower vehicle from a great distance, greater than his desired following distance, the limits of human perception play an important role in adjusting the speed of the driver. The bigger the distance between the driver and the vehicle in front of him, the more difficult it is for the driver to estimate the speed difference of the two vehicles. The driver will adjust his speed as soon as he is capable of correctly estimating the speed of the vehicle in front of him. This physical aspect of the driver is modeled in Fosim according to the Wiedemann model [23]. The important part of the Wiedemann model is the perception threshold, which indicates when a driver adjusts his speed as a consequence of the lower speed of the vehicle in front of him. This treshold depends on the distance between the two vehicles as well as on their speed difference. The principle of the Wiedemann model is graphically shown in Figure 6.

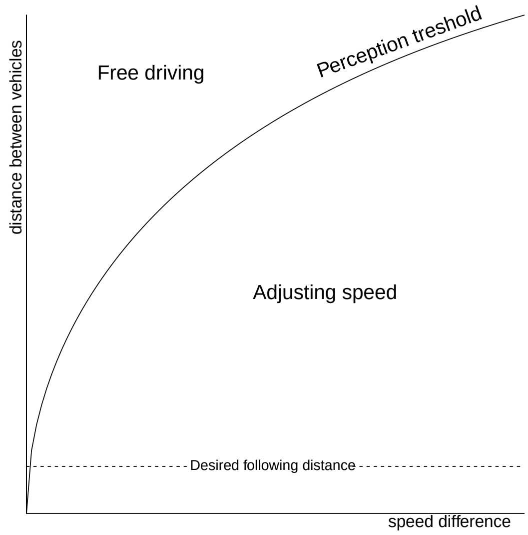


Figure 6: Principle of the Wiedemann model

As can be seen from the figure, when the driver is above the perception threshold, he maintains or accelerates to his desired speed. Conversely, when he is below this threshold, the driver decelerates to decrease the speed difference between both vehicles. This will lead to a process in which the driver accelerates and decelerates to maintain distance close to the desired following distance.

The distance between two vehicles could also be relatively small, i.e. smaller than the desired following distance of the driver. When this is the case, another model is applied in Fosim for the acceleration of the driver. In this model, the acceleration is determined such that the following distance is at least the desired following distance within a certain time period, which is called the anticipation time. Fosim calculates with this anticipation time both the acceleration that is needed to avoid accidents and to attain the desired following distance. The models described in this section are used to simulate the individual behavior of vehicles in Fosim. Based on these models, and with the simulation settings described in the next section, the simulated data can be obtained from Fosim.

3.3 Simulation settings

In order to simulate the Fosim data that will be used for the emulation of the relevant uCAN data, the choices of the road layout and the traffic characteristics in the simulation model should be specified. The former will be discussed in Section 3.3.1, where different choices are made such as the number of lanes and the lane types. The traffic characteristics are divided into two sets of parameters, namely the fixed calibrated parameters, described in Section 3.3.2, and the free parameters, described in Section 3.3.3. After the road layout is defined and the parameters are set, the simulation can be run. To obtain the Fosim data that will be used to emulate the uCAN data from the simulation, inductive loop detectors are placed at the road section. The settings for these detectors will be explained in Section 3.3.4.

3.3.1 Road layout

For the road design in Fosim, a two-lane highway, where it is allowed to change langes, with one entrance ramp is chosen, all for one direction of the traffic. The choice for a two-lane highway is based on the fact that each highway has at least two lanes. The entrance ramp is placed so that congestion can be simulated. Furthermore, the maximum allowed speed of this simulated highway is set equal to 120 km/h, which is the standard maximum speed of the highway in Fosim. The length of the highway is equal to 1750 metres, and the entrance ramp, starting at 500 metres, joins the main highway at 1000 metres. It is chosen that there is one location where the vehicles leave the highway, so there is only one destination. The layout of the road design is shown in Figure 7.

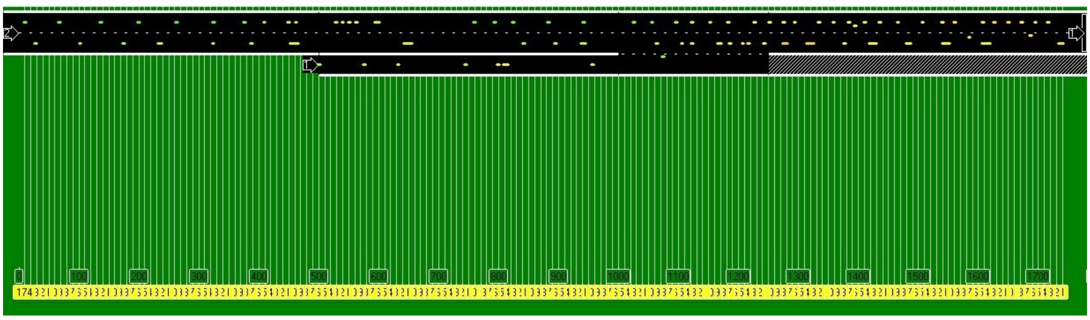


Figure 7: Road layout in Fosim used for simulations

As can be seen from Figure 7, the vehicles are spread over the road, where the gaps between successive vehicles differ. This is a result of the random vehicle generator and the individual behavior defined by the models described in Section 3.2. Furthermore, there are dots and short lines on the highway, representing cars and freight carriers respectively. These different types of vehicles will be described in the next section. The vertical lines shown below the road section represent the inductive loop detectors, which will be discussed in Section 3.3.4. Moreover, as can already be seen from the figure above, the vehicles have different colors. These colors indicate the speed of the vehicles, where green means that the vehicle is driving with a speed of 120 km/h, while yellow means a speed of 80 km/h. Since there is no congestion in this figure, vehicles with low speeds are not shown. In Figure 8 below an example is given of congestion, which is shown by the line of dark red dots representing vehicles with a speed of 0 km/h.

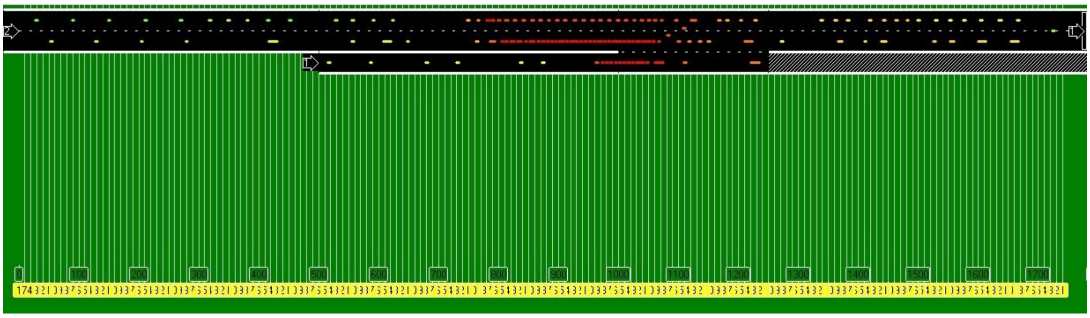


Figure 8: Road layout with congestion in Fosim used for simulations

Note that also lane changes are graphically shown, by dots moving from one lane to the other lane, see the two vehicles at the end of the main highway and the vehicle entering the main highway from the entrance ramp for example in Figure 7. The simulation results depend on this road layout, and also on the parameters of the traffic characteristics, which will be discussed in the next two sections.

3.3.2 Calibrated parameters

The first set of traffic characteristics are the fixed calibrated parameters. Calibrated parameters define properties like the maximum acceleration of the vehicle or the parameters for the lane change behavior of the driver. As mentioned above, Fosim is specifically developed for the highways in the Netherlands. This has resulted in fixed parameters for five different types of vehicles that are validated and calibrated for the Dutch highways [24]. The different types of vehicles consist of three types of cars and two types of freight carriers. These various types of vehicles distinguishes the different behavior of different drivers. For example, the maximum speed of the simulated highway is 120 km/h. In this case, the cars of type 1 has a desired speed of 125 km/h, while the cars of type 2 and 3 have a desired speed 115 km/h and 100 km/h respectively. Freight carriers have a lower desired speed, since their maximum allowed speed is 80 km/h. The freight carriers of type 1 has a desired speed of 95 km/h, while the freight carriers of type 2 has a desired speed of 85 km/h. The values of the other fixed parameters can be found in [18]. In the next section, the free to choose parameters are discussed.

3.3.3 Free parameters

The second set of parameters that define the traffic characteristics contains the parameters that differ through the day. This set consists of three parameters, namely the traffic composition, the traffic intensity and the duration of the simulation. Regarding the latter parameter, it is chosen to run the simulation for one full day of 24 hours, i.e. 1440 minutes, since then situations such as congestion can be observed. The traffic composition is defined by the percentage of freight carriers on the road section. Since freight carriers are one of the major users of the highways, it is important to include this type of vehicle in the simulations. Based on [25], the percentage is chosen to be 15%, which means that on average 15% of the vehicles on the road section will be a freight carrier. The last free parameter, the traffic intensity, is defined on multiple time intervals of the simulation, which means that a different value of the intensity can be assigned to different time intervals. As a result, congestion can be simulated by choosing the traffic intensity parameters close to the capacity, see Chapter 2 for the capacity of a two-lane highway with one entrance ramp. When the intensity is close to this capacity for a longer period of time, there will be too many vehicles on the road section, which will lead to a congestion. By giving a low value to the intensity for the subsequent time interval, the congestion will disappear. In Table 1 below, an example of the intensities per time interval is given.

Time (s)	0	900	2400	3600	3825	4050	4500	5400
Intensity main highway (veh/h)	2100	3600	2000	2100	1900	3550	2300	2250
Intensity entrance ramp (veh/h)	800	1300	450	800	400	900	950	800

Table 1: Intensities given in Fosim.

The intenstities given in the table above are the intensities at the corresponding times, for the first 90 minutes of a simulation. The intensity at the times in between the mentioned times are determined by linear interpolation of the given times and intensities. The intensity on the main highway is set equal to a relatively high value at 900 seconds and 4050 seconds. Together with a relatively high intensity at the entrance ramp, this will result in a congestion.

When the intensity is very low, the road section is almost empty, so all vehicles can drive their desired speed and the traffic speed will be close to the maximum allowed speed. For relatively high intensities, interaction between vehicles takes place. In this situations, which could lead to congestion, the traffic speed can be lower than the maximum allowed speed. This means that in this situations the traffic speed plays an important role in order to manage the traffic and to reduce congestion. Therefore, the intensities where the traffic speed differ relatively much from the maximum allowed speed are considered in this thesis. The lower bound for these intensities is based on the two-second rule. The two-second rule is a rule of thumb for drivers so that they can maintain a safe following distance at any speed. This rule for the distance between two vehicles is used to determine the minimum intensities that are considered. When there are two seconds between each vehicle, the road is crowded. In this situation, $2 \cdot \frac{60}{2} = 60$ vehicles in one minute will pass a certain point on the two-lane highway. To investigate the less crowded situations where the traffic speed starts playing an important role for traffic management, the minimum intensity is chosen as the intensity for which the difference between vehicles is on average 4 seconds. With this difference of time between the vehicles, the total number of vehicles that will pass a certain point on a two-lane highway in one minute is equal to $2 \cdot \frac{60}{4} = 30$ vehicles. This translates to an intensity of $60 \cdot 30 = 1800$ vehicles per hour for the main highway. For this reason, only intensities on the main highway bigger than 1800 are found in Table 1. The resulting minimum total number of vehicles in one minute will then be approximately equal to 30, which will be used in the analysis of Chapter 6.

With the above discussed parameters and the road layout described in Section 3.3.1, the simulation can be run. To obtain the Fosim data from this simulation, inductive loop detectors are placed on the road section. How this is done is described in the next section.

3.3.4 Inductive loop detectors placed in Fosim

The main way to obtain data in Fosim is via inductive loop detectors, which work in a similar way to the ones that are used for real highways, see Section 2.1. An important remark is that, in contrast to detectors that are used in practice, the detectors placed in Fosim do not make measurement errors, so the exact data of a vehicle is given at the place of a detector. The detectors are placed at fixed locations, so the data of each vehicle is sent at the fixed locations in Fosim. However, the uCAN data from a specific vehicle is given each second and thus not at a fixed places. Since it is not possible in Fosim to get information directly from a vehicle once per second, the uCAN data cannot be directly obtained from the simulation. The relevant uCAN data needs to be emulated from the data of the inductive loop detectors in Fosim, where an approximation error will be made, since the data can only be obtained at the locations of the detectors and not per second. When the detectors are placed every 500 metres, as in practice, the time that it takes for a vehicle to pass a detector is approximately equal to 15 seconds when the vehicle is driving the maximum allowed speed of 120 km/h, and it even takes a longer time when a vehicle has a lower speed. Since this time is much longer than the time between two moments that uCAN data is collected, i.e. one second, the approximation error will be relatively large. To reduce this approximation error, the detectors in Fosim are placed every 10 metres to obtain more information about each vehicle. With the data from these detectors, also called the Fosim data, the relevant uCAN data will be emulated. In the next section this Fosim data will be described.

3.4 Fosim data

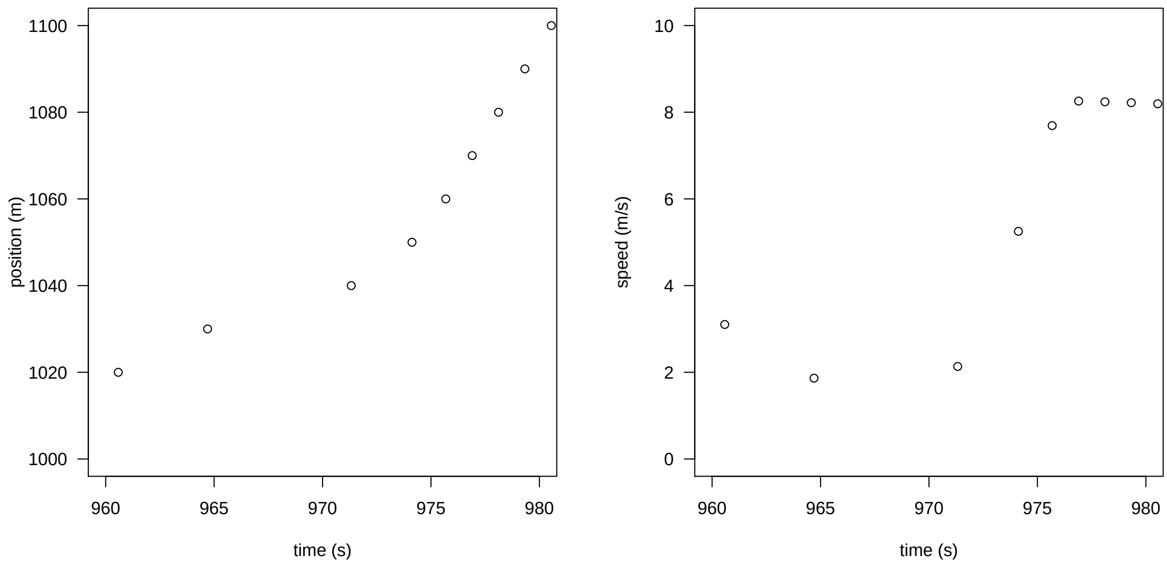
In Section 3.3 the settings in Fosim to obtain Fosim data that will be used to emulate the relevant uCAN data are discussed. The resulting Fosim data can be examined during the simulation and can also be given in an output file. This file contains information about each vehicle that passes an inductive loop detector that is placed at the road section. Table 2 shows three rows of output data from Fosim, where the simulation is run with the settings of Section 3.3.

pos (m)	lane	t (s)	v (m/s)	type	id	dest
340	2	231.082	31.9444	2	24	1
120	1	458.467	23.6111	4	47	1
470	2	482.563	27.7778	1	50	1

Table 2: Fosim data

As can be seen from the table above, there are five variables given in the output file. The variable `pos` in Table 2 indicates the position in metres of the inductive loop detector that was passed by the vehicle. Since the length of the considered highway is equal to 1750 metres and detectors are placed every 10 metres of the road section, it follows that $\texttt{pos} \in \{0, 10, \ldots, 1740, 1750\}$. A detector that is placed at a certain position measures the data at every lane on that position. The variable `lane` gives the number of the lane where the vehicle passes the detector. Because the simulated highway is a two-lane highway with one entrance ramp, `lane` is equal to 1, 2 or 3. The variable `t` represents the time in seconds at which the vehicle passes the detector. The simulation has ran for one day, which means that $\texttt{t} \in [0, 86400]$. Moreover, for each vehicle, the value of `t` is bounded by the time that the vehicle passes the first detector, denoted by t_{min}, and by the time that the vehicle passes the last detector, denoted by t_{max}. Another variable is the speed of the vehicle at the moment that the vehicle passes the detector. This variable is given by `v` and is measured in metres per second. The value of `v` can be between 0, in case of congestion where the vehicles are not driving, and 35. This means that the maximum speed of a vehicle is equal 126 km/h. The variable `type` indicates the type of a vehicle that passes the detector and is already described in Section 3.3.3, so `type` $\in \{1, 2, 3, 4, 5\}$. Furthermore, every vehicle in Fosim has a unique identification number. On the basis of this number, given by `id`, the same vehicle is retrievable at every detector. During the simulation, in total 76257 different vehicles have passed at least one detector, so `id` $\in \{1, 2, \ldots, 76256, 76257\}$. The last variable in the table is `dest`, which shows the specified destination of each vehicle. Because there is only one destination specified in the simulation settings of Fosim, `dest` can only take on the value 1.

As mentioned above, the relevant uCAN data are the positions and the speeds at corresponding times of each vehicle. This means that only the variables `pos`, `t`, `v` and `id` of the Fosim data are of interest in this thesis. Since the detectors are placed at fixed locations, the Fosim data is sent at fixed locations. Because most vehicles are driving with different speeds, the time steps of the data in Fosim are not fixed. For example, suppose that two detectors are placed at the road section at `pos` $= 300$ and `pos` $= 330$, and that two vehicles are passing the first detector at the same time, say `t` $= 150$. If vehicle A drives 120 km/h on the highway and vehicle B drives 80 km/h, then the second detector gives the data at `t` $= 150.9$ and `t` $= 151.35$ respectively, so the time steps are not fixed. This can also be shown graphically by considering one vehicle that drives on the highway. The output file from Fosim contains the positions and speed at the corresponding times of the vehicle. In Figure 9 a part of the data for this vehicle is shown.



(a) Time and corresponding position (b) Time and corresponding speed

Figure 9: Time with corresponding position and speed of the car given by Fosim

Figure 9(a) shows at every ten metres the corresponding time of the vehicle and Figure 9(b) shows the corresponding speed of the vehicle. In Figure 9(b) it can be seen that information of the vehicle is given more often when the vehicle is driving with a higher speed. As a consequence, the position and the speed of the vehicle are not given at fixed time steps in Fosim. However, the uCAN does send the data at fixed time steps, namely once per second, and thus this Fosim data cannot be directly used as emulated uCAN data. In the next chapter, the emulation of the uCAN data with the Fosim data as described in this chapter will be discussed. After that, this emulated uCAN data can be used to estimate the traffic speed.

4 Emulating the uCAN data

The relevant uCAN data for this research are the speed and the associated position of a vehicle, which is given once per second. The data from the output file from Fosim does not directly give this data, it needs to be processed in order to get the relevant uCAN data. From the Fosim data, the speed and the position of a vehicle are known on a discrete grid from t_{min} to t_{max}. Since the speed and the position of a vehicle in Fosim are given every 10 metres, these values are not given every second, because the higher the speed of a vehicle, the shorter the period between two data points. First, to emulate the data of vehicles equipped with a uCAN, which is given every second, the data from Fosim will be interpolated. The speed and position of a vehicle are then known at every arbitrary time $t \in [t_{min}, t_{max}]$. Section 4.1 discusses, after describing different interpolation methods, the interpolation of the Fosim data in detail. Second, the obtained interpolants will be discretised in Section 4.2 to extract the once-per-second data, which represents the uCAN data.

4.1 Interpolation of Fosim data

If real inductive loop detectors were used to emulate the uCAN data, noise should be taken into account, since detectors make measurement errors. Because of this noise, the exact speeds and positions of a vehicle are not known, only approximate values are known. Moreover, the underlying functions of the speed and position are not known as well. To obtain approximations of these functions, the measurements from the detectors can be used to construct a smoothing spline. This spline can be used to fit a curve to a set of noisy observations, where this curve only captures the pattern of the measurements. The curve will therefore not pass through all the observations, which compensates the errors made in the measurements.

In contrast to real inductive loop detectors, the detectors in Fosim do not make measurement errors. This means that the data from Fosim are the exact values of the position and speed functions of a vehicle at the corresponding times. Though, as mentioned above, the underlying functions themselves are not known. To emulate the relevant uCAN data, these functions have to be known. Since the data from Fosim contains the exact speeds and positions, an approximation to the speed and position functions should pass through all the observations. So, instead of approximating the functions with a smoothing spline, the functions can be approximated by spline interpolation of the Fosim data.

To obtain the speed and position of a vehicle at every arbitrary time t, the Fosim data will be interpolated. This means that the underlying function will be approximated based on the known speeds and positions and the corresponding times in the output file from Fosim. To interpolate given knots, i.e. to obtain the exact function values, there are different methods available. Spline interpolation methods are the most commonly used. A spline is a piecewise polynomial function passing through all the knots. Even if each piece is a low-degree polynomial, the spline itself can be a complicated function. Therefore, it can be used for an accurate approximation of the underlying function of almost any set of knots. After spline interpolation, the position and speed of a vehicle are known at each time point t. The resulting interpolant does not only depend on the given knots, but also on the choice of the degree of the polynomial subfunctions of the spline. Note that for some splines the resulting interpolant depends on additional conditions as well.

The best known splines are the linear spline and the cubic spline, which are constituted by first degree and third degree polynomial subfunctions respectively, and will be discussed in Section 4.1.1. In Section 4.1.2, cubic Hermite spline interpolation, which is an interpolation method that is slightly different than cubic spline interpolation, will be discussed. An example of interpolated Fosim data that is used for emulation of uCAN data is given in Section 4.1.3.

4.1.1 Linear and cubic spline interpolation

Consider a set of knots and corresponding function values of these knots $(x_1, y_1), \ldots, (x_n, y_n)$, with $n \in \mathbb{N}$, where y_j is the known value of the underlying function at x_j, for $j = 1, 2, \ldots, n$. These points, i.e. the given data points in Fosim, can be interpolated to approximate the underlying function. The linear spline interpolant of two knots (x_i, y_i) and (x_{i+1}, y_{i+1}) is the straight line connecting these points, so for $x \in [x_i, x_{i+1}]$ the corresponding value of the i^{th} subfunction f of the interpolant is then equal to

$$f(x) = \frac{x - x_i}{x_{i+1} - x_i}(y_{i+1} - y_i) + y_i.$$

The linear spline interpolant based on the whole set of points is obtained by connecting the straight lines between each pair of consecutive points, which results in a continuous piecewise linear function.

This method is similar to cubic spline interpolation, where the latter does not have straight lines between each two points but third degree polynomials instead, which are connected at the given knots. As a result, the cubic spline is also a continuous function. The i^{th} subfunction g_i of a cubic spline is therefore of the form

$$g_i(x) = ax^3 + bx^2 + cx + d, \quad \text{for } x \in [x_i, x_{i+1}] \quad \text{and } a, b, c, d \in \mathbb{R}.$$

To determine the subfunctions, the four unknown values a, b, c, d have to be computed. Since the considered set is a collection of n knots, the cubic spline has $n - 1$ subfunctions. For these $n - 1$ intervals, there are $4(n - 1) = 4n - 4$ unknown variables. From the function values at the knots, $2(n-1)$ equations for these unknowns are given. For the i^{th} subfunction, these equations are

$$g_i(x_i) = y_i \qquad g_i(x_{i+1}) = y_{i+1}. \tag{1}$$

Since the number of equations is less than the number of unknowns, the cubic spline is not uniquely determined by these function values. It is therefore required that the spline is twice continuously differentiable, which means that the first and second derivative of this spline are continuous, so

$$\begin{aligned} g'_{i-1}(x_i) &= g'_i(x_i) \qquad & g'_i(x_{i+1}) &= g'_{i+1}(x_{i+1}) \\ g''_{i-1}(x_i) &= g''_i(x_i) \qquad & g''_i(x_{i+1}) &= g''_{i+1}(x_{i+1}). \end{aligned} \tag{2}$$

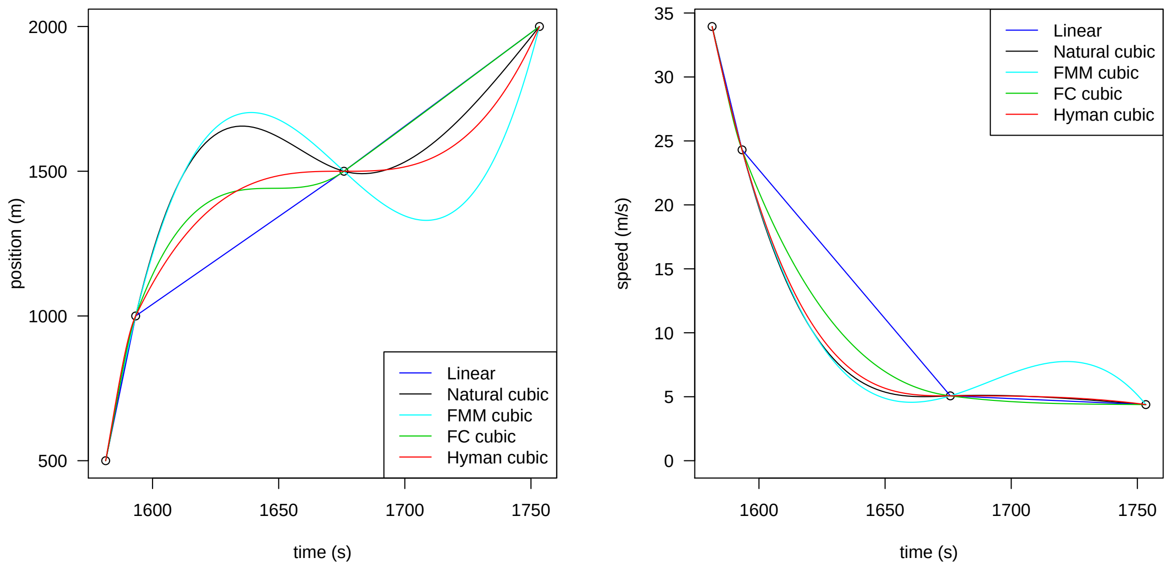
These requirements impose $2(n - 2)$ equations, so the total number of $4n - 6$ equations is then given for $4n - 4$ unknowns. Even with these requirements the cubic spline is therefore not unique given only the function values of the knots. To uniquely define a cubic spline, two additional conditions have to be imposed. There are several well-known conditions, which lead to different types of cubic splines. For example, the natural cubic spline is defined by the additional conditions that the second derivatives at both end knots are equal to zero, i.e.

$$g''_1(x_1) = 0 \qquad g''_{n-1}(x_n) = 0. \tag{3}$$

The given function values (1) together with the requirements (2) and the two conditions (3) constitute $4n-4$ equations which uniquely define the $4n-4$ unknown variables and thus impose a unique cubic spline.

To show the resulting interpolants of the linear and the cubic spline methods, an example data set from Fosim is used. In practice, a common distance between two inductive loop detectors is 500 metres. Therefore, in this example a total of four detectors, with a distance of 500 metres between successive detectors, are placed on a two-lane highway. This example is used to clarify the differences between the discussed interpolation methods on the basis of the relatively large distance between two knots. Note that for emulation of the uCAN data the inductive loop detectors in Fosim are placed every 10 metres, so the distances between the knots used for this emulation are relatively small. As a result, the approximation with spline interpolants of the underlying functions is more accurate when knots are given every 10 metres.

For the example with four detectors placed every 500 metres, the exact speeds, positions and corresponding times at these knots of only one vehicle are used for interpolation. With these knots, the position and speed functions are approximated with linear and cubic spline interpolants. For the latter interpolant, four different cubic splines are chosen, namely the natural cubic spline, the Forsythe, Malcolm and Moler (FMM) cubic spline, the Fritsch and Carlson (FC) cubic spline and the Hyman cubic spline. The resulting spline interpolants of the knots, which approximate the position and speed functions, are shown in Figure 10, where the dots represent the knots. The additional conditions of these interpolants are described below the figure.



(a) Spline interpolants of the position function (b) Spline interpolants of the speed function

Figure 10: Spline interpolants

The additional conditions of the natural cubic spline are given in (3). The FMM cubic spline interpolant is determined by considering the unique third degree polynomials p_1 and p_2 passing through the first four knots and the last four knots respectively. The interpolant is then uniquely defined by setting the third derivative of the interpolant at the first and last knot equal to the third derivative of p_1 and p_2 at the corresponding knots respectively. For the construction of a FC cubic spline interpolant, the algorithm described in [27] can be used. In this algorithm, the derivatives at the knots are computed such that, if and only if the function values at the knots are monotone, monotonicity is ensured. The Hyman cubic spline interpolant can only be applied if the function values are monotone. This interpolant is constructed by choosing the derivatives at the knots as described in [28]. Similar to FC cubic spline, this scheme for computing the derivatives at the knots results in a monotone interpolant. Based on Figure 10, the choice of the interpolants for both underlying functions will be discussed.

In Figure 10(a), the above mentioned spline interpolants for the position knots are shown. As can be seen from this figure, the linear spline interpolant is not a smooth function. The derivatives between the knots are constant, while at the knots the function is not differentiable. Since the derivative of the position with respect to time is the speed, the linear spline interpolant of the position knots results in a speed function of the car that is piecewise constant and, unless it is a constant function on the whole support, therefore not continuous. This means that the speed changes instantaneously when the vehicle passes a detector. In reality, this is not possible, so the linear spline results in an unrealistic approximation of the position function. For this reason, the linear spline is not used for the interpolation of the position knots of the Fosim data. Since the derivative of the speed, which is the accelaration of the vehicle, cannot change instantaneously either, the linear spline of the speed knots shown in Figure 10(b) will neither be used for the interpolation of these knots.

Figure 10 also shows the cubic spline interpolants. Compared to the linear spline, these splines have the advantage that they are (twice) continuously differentiable. Moreover, the different additional conditions affect the entire curve of the interpolants, which result in different cubic splines. For the speed knots, shown in Figure 10(b), the interpolants are very similar, except for the FMM cubic spline. The other three interpolants do not only agree on the last interval, but are also similarly shaped on the other intervals. However, the Hyman cubic spline interpolant can only be constructed for monotonic data. Although the speed knots in the example are monotonic and thus can be interpolated with the Hyman cubic spline, in general the speed knots will not be monotonic. Because of the similarity between the other two interpolants, either one of these two interpolants will be chosen. Since the most common cubic spline is the natural cubic spline, this interpolant is chosen to approximate the underlying speed function.

When the position knots are considered, a different choice for the interpolant of this underlying function is made. Two cubic splines of Figure 10(a), namely the natural cubic spline and the FMM cubic spline, decrease on a certain interval of the domain. Since the distance between the inductive loop detectors in this example is relatively large, the decreasing property of both cubic splines for the interpolation of the position knots can be clearly seen in the figure. This negative trend of both curves means that the vehicle is driving backwards on the highway, which in general cannot happen. These cubic splines are therefore not chosen to interpolate the position knots. The other two cubic splines, i.e. the FC and Hyman cubic spline, are monotonically increasing. However, the disadvantage of these splines is that they, especially in the third interval, differ much from each other. Which one of the splines to choose is not directly clear, so the interpolant that will be used for the position knots is not selected yet. There is more information available, though, that can be used to approximate the position function more accurate. As mentioned above, also the exact speeds of the vehicle at the knots are known. These speeds are the first derivatives of the position function and thus give more information about this function. However, none of the spline interpolants discussed above make use of this extra information. There is a slightly other method than cubic spline interpolation, namely cubic Hermite spline interpolation, that does make use of these speeds for the construction of the interpolant and can therefore be used for a more accurate interpolation of the position knots. This adapted version of cubic spline interpolation will be explained in detail in the next section.

4.1.2 Cubic Hermite spline interpolation

As mentioned above, a cubic spline is defined by a piecewise third degree polynomial function. Furthermore, for a given set of knots and corresponding function values at these knots $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, with $n \in \mathbb{N}$, the i^{th} subfunction of the interpolant is specified by the values (x_i, y_i) and (x_{i+1}, y_{i+1}) and the previously described requirements (2) and conditions (3). This interpolation method does not make use of the extra information, namely the first derivatives at the knots, that is available. To include these first derivatives, the cubic Hermite spline interpolant will be considered. Each subfunction of a cubic Hermite spline is also defined by a third degree polynomial function, however, this polynomial function is not only specified by the points (x_i, y_i) and (x_{i+1}, y_{i+1}), but also by the first derivatives at these knots, denoted by z_i and z_{i+1}. In our situation, x_i represents the time, y_i the position and z_i the corresponding speed of the vehicle.

Let (x_j, y_j, z_j), for $j = 1, 2, \ldots, n$ be given. The i^{th} subfunction, denoted by h, of the cubic Hermite spline is then defined on the interval $[x_i, x_{i+1}]$ and, because it is a third degree polynomial function, can be written as

$$h(x) = ax^3 + bx^2 + cx + d, \quad \text{for } x \in [x_i, x_{i+1}].$$

The constants $a, b, c, d \in \mathbb{R}$ need to be computed to obtain the polynomial function for this interval. The knots can be translated to the following equations for h:

$$\begin{aligned} h(x_i) &= y_i \qquad & h(x_{i+1}) &= y_{i+1} \\ h'(x_i) &= z_i \qquad & h'(x_{i+1}) &= z_{i+1}. \end{aligned} \tag{4}$$

Since there are four unknown constants and four equations for these constants, only the i^{th} interval has to be considered to uniquely determine the i^{th} subfunction of the cubic Hermite spline. Then, by combining the subfunctions, the cubic Hermite spline interpolant is uniquely defined. Any assumptions about continuity of the second derivative can therefore not be made. As a consequence, in contrast to the cubic spline, the second derivative will in general not be continuous. The first derivative, however, is continuous since the derivative of each subfunction is a second degree polynomial function, and these polynomials are connected by the knots (x_i, z_i).

First, the unique i^{th} subfunction of the cubic Hermite spline will be determined, after which the subfunctions of all intervals will be combined to construct the unique cubic Hermite spline interpolant on the whole domain $[x_1, x_n]$. Second, it is shown that this method gives a better result than the interpolants shown in Figure 10.

The subfunction of the i^{th} interval can be constructed by solving equations (4) for the constants a, b, c, d. These equations can be expressed in terms of the constants, which leads to the following system of equations:

$$
\begin{aligned}
ax_i^3 \quad &+ \quad bx_i^2 \quad + cx_i \quad + d = y_i \\
ax_{i+1}^3 &+ \quad bx_{i+1}^2 + cx_{i+1} + d = y_{i+1} \\
3ax_i^2 \quad &+ 2bx_i \quad + c \qquad\quad = z_i \\
3ax_{i+1}^2 &+ 2bx_{i+1} + c \qquad\quad = z_{i+1}.
\end{aligned}
$$

One way to solve this system is to transform the given knots. Define the transformed variable $\tilde{x}$ as

$$\tilde{x} = \frac{x - x_i}{x_{i+1} - x_i}. \tag{5}$$

Since $x \in [x_i, x_{i+1}]$, it holds that $\tilde{x} \in [0, 1]$. The transformed i^{th} subfunction $\tilde{h}$, which is thus defined on the unit interval, can be expressed as $\tilde{h}(\tilde{x}) = \tilde{a}\tilde{x}^3 + \tilde{b}\tilde{x}^2 + \tilde{c}\tilde{x} + \tilde{d}$, with $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d} \in \mathbb{R}$. The transformation (5) is only a horizontal transformation, so the function values do not change, and thus $\tilde{h}$ can be related to h by $\tilde{h}(\tilde{x}) = h(x)$. This leads to the following equations for the function values $\tilde{y}_i$ and $\tilde{y}_{i+1}$ of $\tilde{h}$ in 0 and 1 respectively:

$$
\begin{aligned}
\tilde{y}_i \quad &= \tilde{h}(0) = \quad h(x_i) \quad = y_i \\
\tilde{y}_{i+1} &= \tilde{h}(1) = h(x_{i+1}) = y_{i+1}.
\end{aligned} \tag{6}
$$

Due to the relationships between $\tilde{x}$ and x and between $\tilde{h}$ and h, the first derivatives of the two polynomials are related by

$$h'(x) = \frac{dh(x)}{dx} = \frac{d\tilde{h}(\tilde{x})}{dx} = \frac{d\tilde{h}(\tilde{x})}{d\tilde{x}}\frac{d\tilde{x}}{dx} = \frac{d\tilde{h}(\tilde{x})}{d\tilde{x}}\frac{1}{x_{i+1} - x_i} = \tilde{h}'(\tilde{x})\frac{1}{x_{i+1} - x_i}.$$

So the derivative is scaled as $\tilde{h}'(\tilde{x}) = (x_{i+1} - x_i)h'(x)$ and therefore the values $\tilde{z}_i$ and $\tilde{z}_{i+1}$ of the first derivative of $\tilde{h}$ in 0 and 1 respectively are:

$$
\begin{aligned}
\tilde{z}_i \quad &= \tilde{h}'(0) = \quad (x_{i+1} - x_i)h'(x_i) \quad = (x_{i+1} - x_i)z_i \\
\tilde{z}_{i+1} &= \tilde{h}'(1) = (x_{i+1} - x_i)h'(x_{i+1}) = (x_{i+1} - x_i)z_{i+1}.
\end{aligned} \tag{7}
$$

The system of equations for the transformed i^{th}subfunction constituted by (6) and (7) can be written as the matrix equation involving the unknowns $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}$

$$
\begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{pmatrix}
\begin{pmatrix} \tilde{a} \\ \tilde{b} \\ \tilde{c} \\ \tilde{d} \end{pmatrix}
=
\begin{pmatrix} \tilde{y}_i \\ \tilde{y}_{i+1} \\ \tilde{z}_i \\ \tilde{z}_{i+1} \end{pmatrix},
$$

which can be solved with the following matrix transformation:

$$
\left(\begin{array}{cccc|c} 0 & 0 & 0 & 1 & \tilde{y}_i \\ 1 & 1 & 1 & 1 & \tilde{y}_{i+1} \\ 0 & 0 & 1 & 0 & \tilde{z}_i \\ 1 & 0 & 0 & 0 & 2\tilde{y}_i - 2\tilde{y}_{i+1} + \tilde{z}_i + \tilde{z}_{i+1} \end{array}\right)
\sim
\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2\tilde{y}_i - 2\tilde{y}_{i+1} + \tilde{z}_i + \tilde{z}_{i+1} \\ 0 & 1 & 0 & 0 & -3\tilde{y}_i + 3\tilde{y}_{i+1} - 2\tilde{z}_i - \tilde{z}_{i+1} \\ 0 & 0 & 1 & 0 & \tilde{z}_i \\ 0 & 0 & 0 & 1 & \tilde{y}_i \end{array}\right).
$$

The polynomial function on the unit interval can then be expressed as

$$\tilde{h}(\tilde{x}) = (2\tilde{y}_i - 2\tilde{y}_{i+1} + \tilde{z}_i + \tilde{z}_{i+1})\tilde{x}^3 + (-3\tilde{y}_i + 3\tilde{y}_{i+1} - 2\tilde{z}_i - \tilde{z}_{i+1})\tilde{x}^2 + \tilde{z}_i\tilde{x} + \tilde{y}_i.$$

By substituting the relations between the transformed knots and the observed knots (6) and (7) in the expression for $\tilde{h}$, the polynomial function h is constructed. As a result, the i^{th} subfunction of the cubic Hermite spline for the observed knots is, after rearrangement of the terms, given by

$$\begin{aligned} h(x) &= \Big(2y_i - 2y_{i+1} + (x_{i+1} - x_i)(z_i + z_{i+1})\Big) \left(\frac{x - x_i}{x_{i+1} - x_i}\right)^3 + \\ &\quad \Big(-3y_i + 3y_{i+1} - (x_{i+1} - x_i)(2z_i + z_{i+1})\Big) \left(\frac{x - x_i}{x_{i+1} - x_i}\right)^2 + \ z_i(x - x_i) + y_i, \end{aligned}$$

with $x \in [x_i, x_{i+1}]$. The resulting function h is only one piece of the cubic Hermite spline interpolant. To construct the interpolant for $t \in [x_1, x_n]$, the above procedure is applied to each interval separately, which leads to the subfunctions $h_1, \ldots, h_{n-1}$ for the $n - 1$ intervals. All subfunctions together form the interpolant H of the knots (x_j, y_j, z_j):

$$H(t) = \sum_{k=1}^{n-1} h_k(t) \mathbb{1}_{[x_k, x_{k+1}]}(t),$$

where $\mathbb{1}_{[x_k,x_{k+1}]}(t) = 1$ if $t \in [x_k, x_{k+1}]$ and $\mathbb{1}_{[x_k,x_{k+1}]}(t) = 0$ if $t \notin [x_k, x_{k+1}]$.

Note that this interpolant cannot be used for the speed knots, since the accelerations of the vehicle are not given, and thus the needed first derivatives of the speed knots are not known. The underlying function of the speed knots is already approximated with the natural cubic spline, so only the position knots are interpolated with the cubic Hermite spline. By applying this interpolation method to the example data set from Fosim described in Section 4.1.1, including the speed knots, the cubic Hermite spline interpolant of the position knots is obtained. The result is shown in Figure 11.

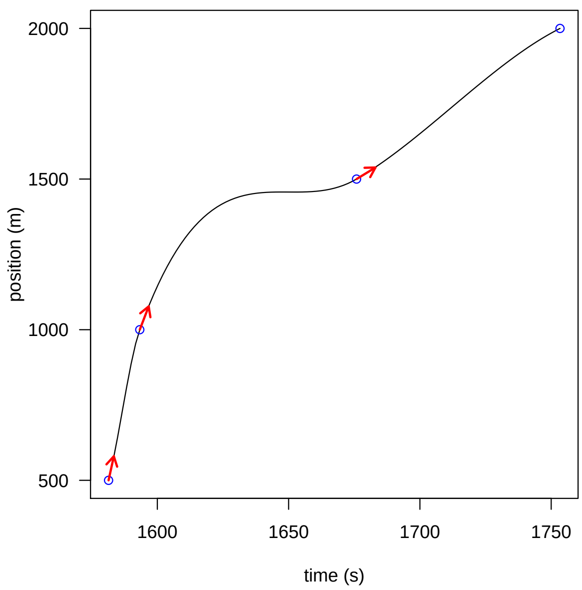


Figure 11: Cubic Hermite spline interpolant

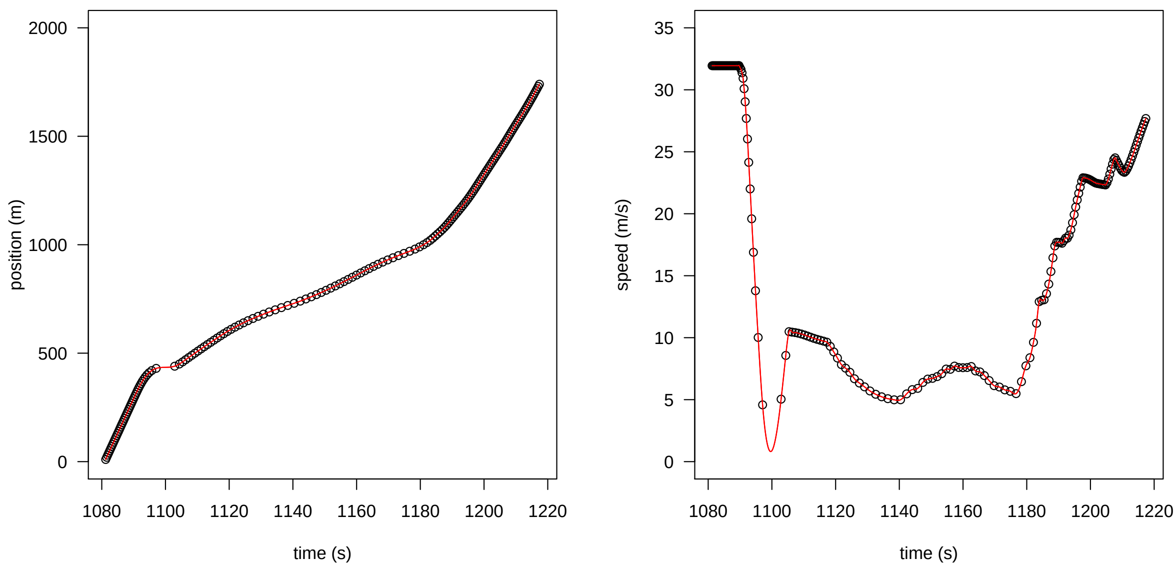
In Figure 11, the arrows represent the given speeds of the vehicle. It can be seen from the figure that the vehicle drives relatively fast at the beginning of the interval, after which it decreases its speed to zero due to a congestion. After some time, the traffic starts moving again and the vehicle slowly increases its speed. The figure shows a realistic result, because the interpolant is an increasing and once continuously differentiable function, with its derivatives in the knots equal to the speed knots. In these aspects the cubic Hermite spline of this example outperforms cubic spline and linear spline interpolation.

However, the cubic Hermite spline is not necessarily an increasing function. Although the FC cubic spline and the Hyman cubic spline are always monotonically increasing for monotone function values, these spline interpolation methods do not use the known information about the speeds. As a result, these interpolants have different derivatives at the knots than the given speeds, which conflicts with reality. Moreover, since the position at the knots are monotone and the derivatives at these knots are positive, it rarely occurs that the resulting cubic Hermite spline interpolant is not monotonically increasing. For these reasons, the cubic Hermite spline interpolant is used in this thesis to approximate the underlying function of the position knots. To ensure that the interpolant of the position knots is monotonically increasing, the subfunction of the cubic Hermite spline on the few intervals where the interpolant is decreasing, is replaced by the linear interpolant between the position knots of that interval.

Note that the derivative of the cubic Hermite spline interpolant of the position knots can be used as the interpolant of the speed knots. However, this derivative is in general not differentiable and, since the acceleration of the vehicle cannot change instantaneously, will therefore not be used to approximate the underlying speed function. As mentioned above, the natural cubic spline interpolant will be applied to the speed knots. In the next section, the Fosim data that is used to give an answer to the main question of this thesis will be interpolated.

4.1.3 Example of the interpolation of Fosim data

The data from Fosim that will be used to emulate uCAN data is different than the example data used in the previous sections. Instead of inductive loop detectors placed every 500 metres, the Fosim data is obtained by placing detectors every 10 metres for more information about the position and speed of a vehicle. Consider the following figure:



(a) Cubic Hermite spline interpolant of position knots (b) Natural cubic spline interpolant of speed knots

Figure 12: Interpolants of knots

As mentioned in the above sections, the underlying function of the position knots is approximated with the cubic Hermite spline interpolant, and the underlying function of the speed knots is approximated with the natural cubic spline interpolant. Figure 12 shows both interpolants of the knots of one vehicle on the entire road section, where the dots represent the Fosim data.

As can been seen from both figures, the road section is crowded most of the time that the vehicle drives on it. When the vehicle starts to drive on the road section, it is driving relatively fast before it suddenly has to break. Figure 12(a) shows that it takes a relatively large time for the vehicle to drive from 430 metres to 440 metres, which is seen by the large time distance between the two knots at approximately 1100 seconds. This can be also be seen from Figure 12(b), because the speed of the vehicle is small near 1100 seconds. After this break, the speed of the vehicle increases again with a break occasionally, since the road section is still crowded. Although the interpolant of the speed knots from 12(b) is two times continuously differentiable, the interpolant is quite rough and seems to have unsmooth bends. Also the speed knots obtained from the Fosim data vary much. This is a result of the models within Fosim, which lead to this behavior of the vehicle. A vehicle will maintain his desired speed as long as it can, and it will accelerate to its desired speed as soon as it can. For this decision, the vehicle do not take into account that it is likely that it will have to break again a short period of time later, which causes the bends that seem to be unsmooth.

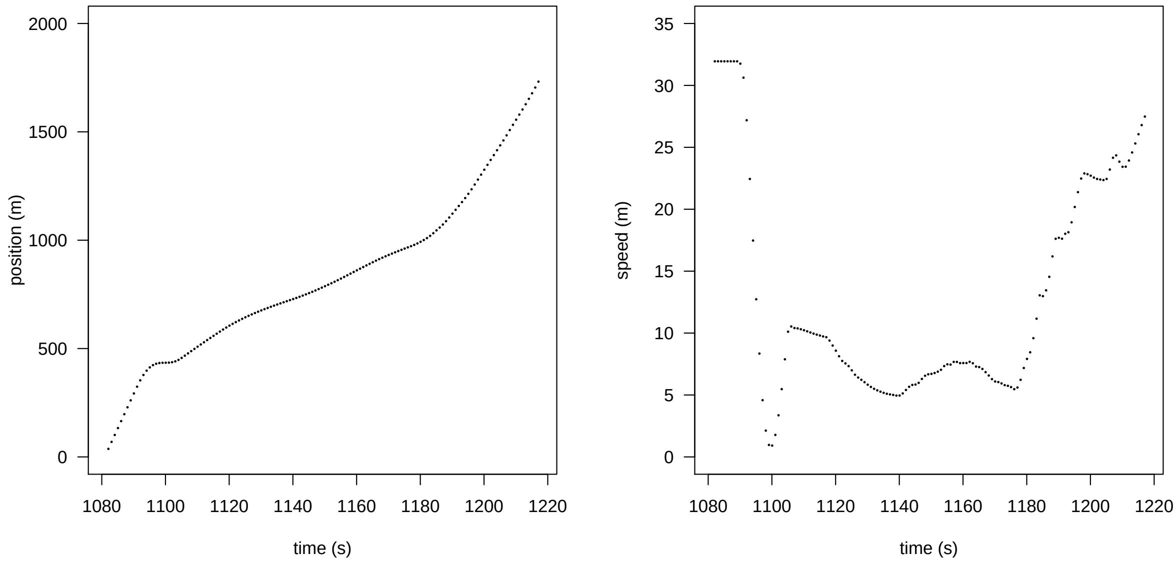
The natural cubic spline interpolant of the speed knots in this example is positive for all t. This interpolant can become negative, though, which would have happened if all speeds of the example were reduced with 3 m/s for example. The other cubic spline interpolation methods can result in interpolants that are negative on certain intervals as well. However, it rarely occurs that the natural cubic spline interpolant becomes negative, since this only happens when a vehicle suddenly reduces its speed to (almost) zero and accelerates right after, similar to the beginning of the speed knots in Figure 12(b). For the intervals that the interpolant is negative, the subfunction of the natural cubic spline will be replaced by the linear interpolant between the speed knots of that interval.

Now that the Fosim data is interpolated, the position and speed of a vehicle are known for each t that the vehicle is driving on the road section. The interpolants of both knots will be used in the next section to emulate the uCAN data.

4.2 Discretization of interpolated Fosim data

The obtained interpolant gives the position and speed of a vehicle at every arbitrary time $t \in [t_{min}, t_{max}]$. However, the uCAN data does not contain all this information, but only the position and speed per second. The second step to emulate the uCAN data is, therefore, to discretize the interpolants from Figure 12. Each vehicle with a uCAN gives the data once per second, but not at the same moment. For example, a vehicle gives the data at $0.4, 1.4, \dots$ seconds, while another vehicle gives the data at $0.7, 1.7, \dots$ seconds. To get these different equidistant times, only the first and the last knot of each vehicle are important. The first time that a vehicle is detected on the road is at 10 metres. A uniform random number between 0 and 1 is added to the time of this first knot to get random first times of sending uCAN data for each vehicle. The uCAN data is then known at this time and at each second later, but not after the time of the last data point. This discretized time represents the equidistant times at which a vehicle sends the uCAN data. Since the position and speed are known, after interpolation, at every arbitrary time, they are also known at the times of the discretization. The discretized time with the corresponding position and speed represents the uCAN data.

As an example, the interpolants from Figure 12 are discretized. The time of the first knot and the time of the last knot of the Fosim data of the considered vehicle are equal to 1081.23 and 1217.39 respectively. A uniform number 0.85 is generated and added to the time of the first knot. The times at which the positions and the speeds of the vehicle are sent then become $1082.08, 1082.93, \ldots, 1216.38, 1217.23$. The corresponding positions and speeds are given by the interpolants. The emulated uCAN data is shown in Figure 13.



(a) Discretized interpolant of position knots

(b) Discretized interpolant of speed knots

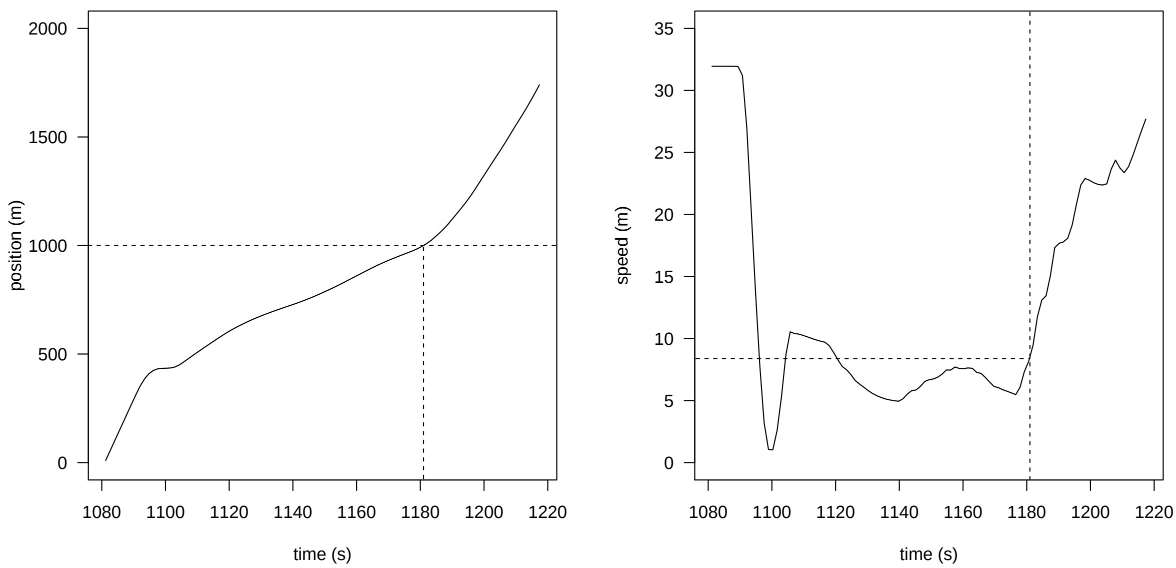
Figure 13: Emulated uCAN data

From Figure 13(a) it can be seen that there are many more data points between position 0 and 300 than between position 400 and 600. This is a consequence of the equidistant discretization of the time, because at a lower speed the vehicle is driving longer on a specific road section, while it keeps sending data every second. This means that a lower speed of a vehicle on a particular position interval leads to more uCAN data in that interval.

4.3 From uCAN data to the traffic speed

In the above sections of this chapter Fosim data is simulated to emulate uCAN data. When in the future the data of vehicles with a uCAN is used to obtain traffic data, this uCAN data does not need to be simulated, since these vehicles will provide their uCAN data. This provided uCAN data can then be used directly for traffic management and thus also to estimate the traffic speed. However, the uCAN data that will be send by vehicles that are equipped with a uCAN, will be received each second. Since the traffic speed is defined at a certain point, the data of the vehicles with a uCAN does not necessarily have to be known at this specific point. Therefore, the uCAN data has to be interpolated.

Because nowadays not enough uCAN data is available to estimate the traffic speed, the emulated uCAN data from Section 4.1.3 will be used instead. For the same reason of the interpolation of the Fosim data, the position data of the above section is interpolated by the cubic Hermite spline, and the natural cubic spline is used as the interpolant of the speed data. This interpolated data can be used to compute the traffic speed. For example, assume that the traffic speed needs to be computed at 1000 metres of the road section in minute 20. To compute the traffic speed in this minute with all the vehicles that pass a certain point in that minute, it is first checked which vehicles pass the considered point in the specific minute. Second, for each of these vehicles, the time that it passes the considered point is determined by the position interpolant, which is shown in Figure 14(a). The speed interpolant can then be used to determine the speed of the vehicle at that time and thus at the point where the traffic speed needs to be calculated, shown in Figure 14(b).



(a) Interpolated uCAN positions

(b) Interpolated uCAN speeds

Figure 14: Interpolated uCAN data

It can be seen from Figure 14 that the time that the vehicle that is considered in the previous section, where the uCAN data for this vehicle was emulated in the previous sections, passes the point 1000 metres is equal to 1181 seconds and that the speed of that vehicle at that position is equal to 8.48 m/s. When this is done for each vehicle that passes the 1000 metres in minute 20, the average of the obtained speeds of all those vehicles can be taken. The traffic speed at the considered point for the specific minute is then determined by this average speeds. In the next chapter, this method is used to estimate the traffic speed based on a the situation that not all vehicles that pass the considered point are equipped with a uCAN.

5 Estimation of the traffic speed

As mentioned in Chapter 1, the traffic speed is defined as the average speed of the vehicles that pass a specific point in one minute. The point that is considered, is the location of an inductive loop detector that is placed on the road section in Fosim. In contrast to real detectors used in practice, where measurement errors occur, the detectors placed in Fosim provide the real speeds of the vehicles. The real traffic speed, denoted by μ_{real}, can then be computed as the mean of the speeds obtained by the considered detector of all the vehicles that pass that detector in one specific minute. Instead of using data from this detector, data from vehicles equipped with a uCAN can be used to obtain another average speed. When all the vehicles that pass the inductive loop detector in one minute are equipped with a uCAN, the average speed computed as the mean of the speeds from the uCAN data of all these vehicles is denoted by μ. This average speed and the above mentioned real traffic speed are not equal to each other. The reason for this is that, in general, vehicles with a uCAN do not send data exactly at the place of the detector, because data is sent every second, so the real speed of a vehicle does not have to be known at the location of the detector. By interpolating the uCAN data, the real speed of a vehicle at every position on the road section can be estimated. These interpolated speeds, and thus not the real speeds, are all used to calculate μ. Both values of μ_{real} and μ can be calculated and compared by simulating 1440 minutes, i.e. a full day, in Fosim. The histogram in Figure 15 shows the relative difference of μ with respect to μ_{real}.

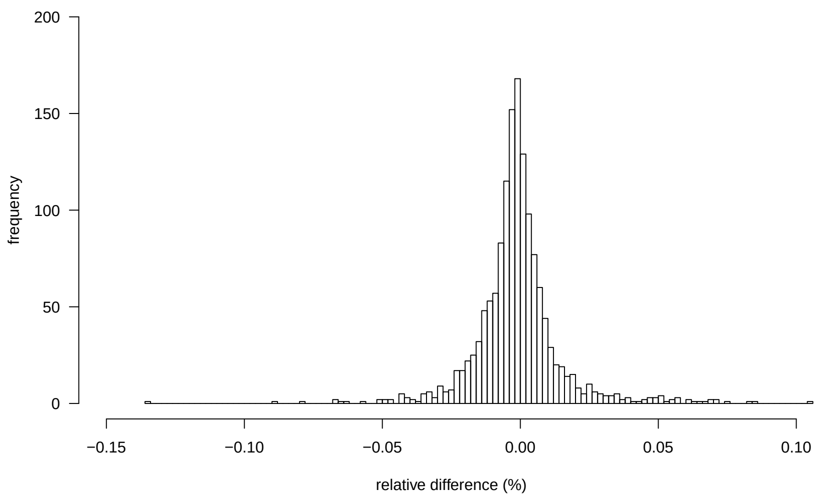


Figure 15: Histogram of $\frac{\mu-\mu_{real}}{\mu_{real}}$

From Figure 15 it can be seen that in 99% of the minutes the relative absolute difference between μ and the real traffic speed is less than 0.065%. From this result it can be concluded that $\mu \approx \mu_{real}$. Because the difference between both average speeds is negligible for practical purposes, μ will be used as the real traffic speed in this thesis.

To calculate μ for a certain point, all vehicles that pass that point in one minute need to be equipped with a uCAN. However, not all vehicles are yet equipped with a uCAN, so the real traffic speed cannot be computed with this method. Though, when a sample of vehicles are equipped with a uCAN, the traffic speed can be estimated with the speeds of the vehicles. In Section 5.1, the estimator of the traffic speed based on the speeds of the vehicles of the sample will be explained in detail and properties of the estimator will be discussed. From these properties, the sampling distribution of the estimator of the traffic speed will be derived. This distribution will be discussed in Section 5.2.

5.1 The estimator of the traffic speed

Consider one minute where a total number of N vehicles pass a certain point, for example in practice an inductive loop detector. The speeds of these vehicles in this minute at the specific point are denoted by $x_1, \ldots, x_N$. The traffic speed is then defined by $\mu = \frac{1}{N}\sum_{i=1}^{N} x_i$ and the variance of the speeds of the N vehicles by $\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$. Of these N vehicles, n ($n \leq N$) vehicles are equipped with a uCAN, so only the speeds of these vehicles are observed. When all the vehicles in the considered minute are equipped with a uCAN, i.e. $n = N$, the traffic speed can be calculated. However, nowadays not all vehicles are equipped with a uCAN, so in practice it will hold that $n < N$. As a consequence, the traffic speed and the speed variance cannot be calculated then.

It is possible, however, to estimate the traffic speed with the average speed of the vehicles with a uCAN. The associated speeds of the total number of N vehicles in the considered minute can be seen as a finite population, with population mean μ and population variance σ^2. The speeds of the vehicles with a uCAN can then be seen as a random sample of size n drawn without replacement from this population. This problem can be related to the statistical model of simple random sampling [29], which can be used to analyse the uCAN data. The speeds of the random sample of size n will be denoted by the identically distributed random variables $X_1, \ldots, X_n$, where $X_i \in \{x_1, \ldots, x_N\}$ for $i = 1, \ldots, n$. The distribution of X_1 is defined by $P(X_1 = x_j) = \frac{1}{N}$ for each $j = 1, \ldots, N$. Note that X_i is the value of the i^{th} element of the sample, which is random, and x_i is the value of the i^{th} element of the population, which is fixed. Because the sample is random, the sample mean, defined by $\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$, is a random variable, of which the expected value can be calculated. Since $X_1, \ldots, X_n$ are identically distributed, it follows that

$$\begin{aligned} E\left[\overline{X}_n\right] &= E\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] = \frac{1}{n}\sum_{i=1}^{n} E\left[X_i\right] = E\left[X_1\right] \\ &= \sum_{j=1}^{N} x_j P\left(X_1 = x_j\right) = \frac{1}{N}\sum_{j=1}^{N} x_j = \mu. \end{aligned} \tag{8}$$

This result shows that $\overline{X}_n$, is an unbiased estimator of μ, so the sample mean is a natural estimate for the population mean. Thus, the mean of the speeds of the vehicles equipped with a uCAN can be used as an estimate of the traffic speed. The quality of this estimator can be assessed by the mean squared error (MSE), which quantifies the difference between the estimated value and the true value. This MSE can be decomposed into contributions from the bias and the variance:

$$\begin{aligned} \text{MSE}\left(\overline{X}_n\right) &= E\left[\left(\overline{X}_n - \mu\right)^2\right] = E\left[\overline{X}_n^2\right] - 2\mu E\left[\overline{X}_n\right] + \mu^2 \\ &= E\left[\overline{X}_n^2\right] - E\left[\overline{X}_n\right]^2 + E\left[\overline{X}_n\right]^2 - 2\mu E\left[\overline{X}_n\right] + \mu^2 \\ &= E\left[\left(\overline{X}_n - E\left[\overline{X}_n\right]\right)^2\right] + E\left[\overline{X}_n - \mu\right]^2 \\ &= \sigma_{\overline{X}_n}^2 + \text{bias}\left(\overline{X}_n\right)^2, \end{aligned}$$

where $\sigma_{\overline{X}_n}$ is the standard deviation, and its square the variance, of the estimator $\overline{X}_n$. Since $\overline{X}_n$ is unbiased, the MSE of the estimator is equal to the variance of this estimator. A smaller

MSE and thus a smaller variance means that the amount by which the estimated value differs from the true value is smaller and that $\overline{X}_n$ is a better estimator of the traffic speed. Therefore, although it is known that on average $\overline{X}_n$ is close to $\overline{X}_N$, it is important to know the variance of $\overline{X}_n$. Analogous to [30], the variance of the estimator of the traffic speed is derived as follows:

$$\begin{aligned}
\sigma^2_{\overline{X}_n} &= \mathrm{Var}\left(\overline{X}_n\right) = \mathrm{Var}\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) \\
&= \frac{1}{n^2}\mathrm{Var}\left(\sum_{i=1}^{n} X_i\right) = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\mathrm{Cov}\left(X_i, X_j\right) \\
&= \frac{1}{n^2}\sum_{i=1}^{n}\mathrm{Var}\left(X_i\right) + \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n}\mathrm{Cov}\left(X_i, X_j\right), \qquad (9)
\end{aligned}$$

with $\mathrm{Cov}\left(X_i, X_j\right)$ the covariance of the two random variables X_i and X_j. Furthermore, since the members of the sample are identically distributed, $X_1, \ldots, X_n$ have the same variance, with

$$\begin{aligned}
\mathrm{Var}(X_1) &= E\left[\left(X_1 - E\left[X_1\right]\right)^2\right] = \sum_{i=1}^{N}\left(x_i - \mu\right)^2 P(X_1 = x_i) \\
&= \frac{1}{N}\sum_{i=1}^{N}\left(x_i - \mu\right)^2 = \sigma^2. \qquad (10)
\end{aligned}$$

So the variance of the speed of a randomly selected vehicle with a uCAN is equal to the variance of the speeds of the N vehicles. To find the variance of $\overline{X}_n$ in sampling without replacement, $\mathrm{Cov}\left(X_i, X_j\right)$ for $i \neq j$ needs to be found. Using the fact that $E\left[X_i\right]E\left[X_j\right] = \mu^2$, the following expression for this covariance is derived for $i \neq j$:

$$\begin{aligned}
\mathrm{Cov}\left(X_i, X_j\right) &= E\left[X_i X_j\right] - E\left[X_i\right]E\left[X_j\right] = E\left[X_i X_j\right] - \mu^2 \\
&= \sum_{k=1}^{N}\sum_{l=1}^{N}\left(x_k x_l P\left(X_i = x_k, X_j = x_l\right)\right) - \mu^2 \\
&= \sum_{k=1}^{N}\sum_{l=1}^{N}\left(x_k x_l P\left(X_j = x_l \mid X_i = x_k\right)P\left(X_i = x_k\right)\right) - \mu^2 \\
&= \sum_{k=1}^{N}\left(x_k P\left(X_i = x_k\right)\sum_{l=1}^{N} x_l P\left(X_j = x_l \mid X_i = x_k\right)\right) - \mu^2 \\
&= \sum_{k=1}^{N}\left(x_k \frac{1}{N}\sum_{l=1, l\neq k}^{N} x_l \frac{1}{N-1}\right) - \mu^2 = \frac{1}{N(N-1)}\sum_{k=1}^{N}\left(x_k \sum_{l=1, l\neq k}^{N} x_l\right) - \mu^2 \\
&= \frac{1}{N(N-1)}\sum_{k=1}^{N}\left(x_k\left(N\mu - x_k\right)\right) - \mu^2 \\
&= \frac{1}{N(N-1)}\sum_{k=1}^{N}\left(-\left(x_k - \mu^2\right) + \mu^2 + (N-2)x_k\mu\right) - \mu^2 \\
&= \frac{1}{N-1}\left(-\sigma^2 + \mu^2 + \mu(N-2)\mu\right) - \mu^2 = -\frac{\sigma^2}{N-1} + \mu^2 - \mu^2 \\
&= -\frac{\sigma^2}{N-1}. \qquad (11)
\end{aligned}$$

The fact that the covariance is negative, implies that if X_i is larger than its expected value, then X_j is likely to be smaller than its expected value. Note that the above described relationship between X_i and X_j is weaker for bigger values of N, since then the covariance is closer to zero. Plugging the above obtained results (10) and (11) into the expression of the variance of $\overline{X}_n$ (9) leads to a relation between the variance of the sample mean and the population variance, namely:

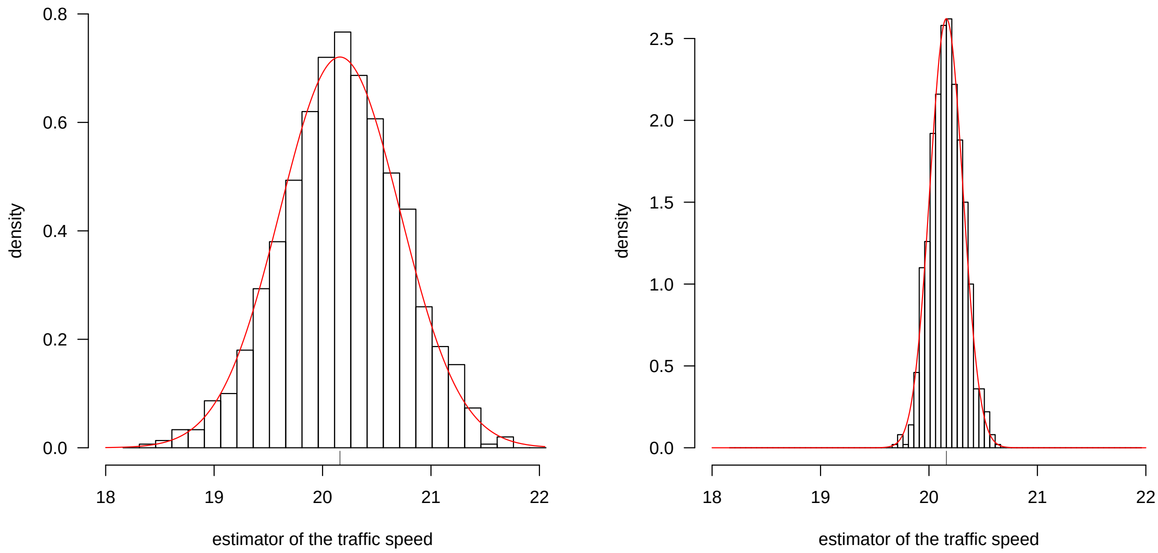
$$\sigma^2_{\overline{X}_n} = \frac{\sigma^2}{n} - \frac{1}{n^2}n(n-1)\frac{\sigma^2}{N-1} = \frac{\sigma^2}{n}\left(1 - \frac{n-1}{N-1}\right) = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right). \tag{12}$$

As can be seen from the result, $\sigma^2_{\overline{X}_n}$ is a decreasing function in n, so a bigger value of n gives a smaller variance of $\overline{X}_n$ and thus a smaller MSE of the estimator. This means that the bigger the value of n, the better the estimator of the population mean, which is intuitively evident as well. Since n is the number of vehicles of the population that is equipped with a uCAN, more vehicles equipped with a uCAN will lead to a better estimator of the traffic speed. The influence of n on $\sigma^2_{\overline{X}_n}$ will be discussed more extensively in the next chapter.

The results of (8) and (12) give the mean and the variance of the estimator of the traffic speed. However, the sampling distribution of this estimator cannot be determined since the population itself is not known. An approximation to the sampling distribution will be deduced in the next section.

5.2 The approximate distribution of the estimator of the traffic speed

To say something about the sampling distribution of $\overline{X}_n$, one minute is considered where a total number of 53 vehicles have passed a certain point. Assume that in this minute 10 vehicles were equipped with a uCAN, so only the speeds of these vehicles are known. With these speeds, the estimator of the traffic speed with $n = 10$ can be calculated. Since the sample of 10 vehicles with a uCAN is random, $\overline{X}_{10}$ is also random. A first idea of the sampling distribution of $\overline{X}_{10}$ is obtained by simulating 1000 realisations of this random variable. Figure 16(a) shows the result.



(a) Distribution of $\overline{X}_{10}$ (b) Distribution of $\overline{X}_{40}$

Figure 16: Distributions of $\overline{X}_n$

The histogram of Figure 16(a) is an estimate of the probability density function of $\overline{X}_{10}$. Furthermore, it is known that the true mean and variance of the considered minute are $\mu = 20.16$ and $\sigma^2_{\overline{X}_{10}} = 0.31$ respectively. So the traffic speed in this minute is 20.16 m/s. The realisations are centered around μ, which is shown by the small vertical line, and all of the 1000 values of the random variable are contained in the interval $[18.33, 21.79]$. So the absolute difference with the true mean is at most equal to 1.75. The red curve is the density function of the normal distribution with mean μ, and variance $\sigma^2_{\overline{X}_{10}}$. As can be seen from the figure, the histogram is close to this density function of the normal distribution.

In Figure 16(b), 40 vehicles were equipped with a uCAN in the considered minute. The histogram in this figure is an estimate of the density function of $\overline{X}_{40}$. As can be seen from the figure, this histogram is also centered around μ, but the histogram is much narrower. All realisations of this estimator are contained in a smaller interval, which is equal to $[19.67, 20.68]$. Therefore, the maximum absolute difference with μ is smaller, so this estimator is more accurate than the estimator for $n = 10$. This is a result from the smaller variance of $\overline{X}_{40}$, which is equal to $\sigma^2_{\overline{X}_{40}} = 0.02$. The difference in the values of the variance of both estimators shows the same conclusion drawn from equation (12), namely that the estimator of the traffic speed is more accurate when more vehicles are equipped with a uCAN. The red curve in this figure is also the density function of the normal distribution with mean μ, but with a smaller variance $\sigma^2_{\overline{X}_{40}}$.

Both histograms of Figure 16 are close to the probability density function of the corresponding normal distribution, suggesting that the estimator is approximately normally distributed. However, this is only an empirical result and not a real proof that the distribution of $\overline{X}_n$ is approximately normal. An approximation to the sampling distribution of $\overline{X}_n$ can be deduced from the central limit theorem for sampling without replacement from a finite population, which is stated below.

Central limit theorem for finite populations
If $X_1, \dots, X_n$ is a simple random sample without replacement from a finite population of size N with population mean μ and population variance σ^2, then it follows that:

$$\frac{\overline{X}_n - \mu}{\sigma_{\overline{X}_n}} \rightsquigarrow \mathcal{N}(0,1) \quad \text{for} \quad n \to \infty \text{ and } N - n \to \infty, \tag{13}$$

where $\sigma^2_{\overline{X}_n} = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right)$.

A proof of this theorem can be found in [31]. The above stated theorem says that $(\overline{X}_n - \mu)\sigma^{-1}_{\overline{X}_n}$ converges in distribution to the standard normal distribution. This means that its distribution function converges to the standard normal distribution function, so for $n \to \infty$ and $N - n \to \infty$

$$P\left(\frac{\overline{X}_n - \mu}{\sigma_{\overline{X}_n}} \leq t\right) \to \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{-\frac{1}{2}s^2} ds, \qquad t \in \mathbb{R}.$$

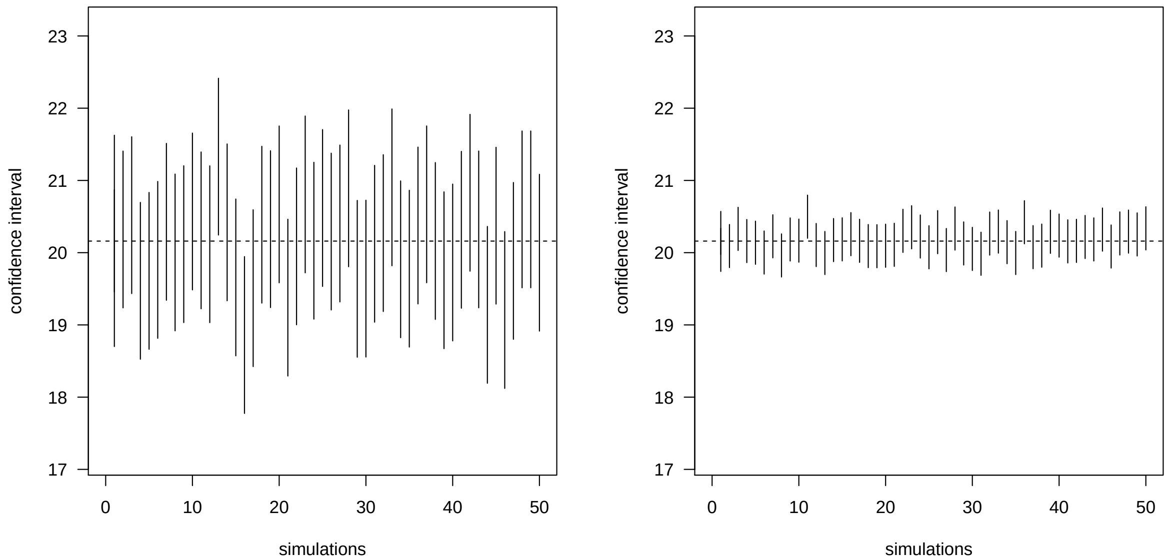
Although this is only a limiting result, the obtained distribution can be used as an approximation of the distribution for finite values of n en N. Since $(\overline{X}_n - \mu)\sigma^{-1}_{\overline{X}_n}$ is thus approximately standard normal, it holds that the distribution of $\overline{X}_n$ can be approximated by the $\mathcal{N}(\mu, \sigma^2_{\overline{X}_n})$ distribution. So it can be concluded that the sampling distribution of the estimator of the traffic speed is approximately normal with the given parameters. From the quantiles of this distribution, it follows that

$$P\left(\mu - 1.96\sigma_{\overline{X}_n} \leq \overline{X}_n \leq \mu + 1.96\sigma_{\overline{X}_n}\right) \approx 0.95. \tag{14}$$

Note that the bounds of $\overline{X}_n$ get smaller as the value of n gets bigger, since the variance of the estimator is then smaller. This result is already shown in Figure 16. By rearranging the terms of the approximate equation (14), a approximate confidence interval for μ can be obtained:

$$P\left(\overline{X}_n - 1.96\sigma_{\overline{X}_n} \leq \mu \leq \overline{X}_n + 1.96\sigma_{\overline{X}_n}\right) \approx 0.95,$$

so $\left[\overline{X}_n - 1.96\sigma_{\overline{X}_n}, \overline{X}_n + 1.96\sigma_{\overline{X}_n}\right]$ is an approximate 95% confidence interval for μ. Since for the considered minute above 1000 realisations of the random variable $\overline{X}_{10}$ are simulated, 1000 confidence intervals for μ can be constructed based on these realisations. Figure 17(a) shows the first fifty of these intervals and the horizontal line at $\mu = 20.16$, which is the traffic speed in this minute with $N = 53$.



(a) Confidence intervals of μ with $n = 10$

(b) Confidence intervals of μ with $n = 40$

Figure 17: Confidence intervals of μ

As can be seen from Figure 17(a), two of the first fifty intervals do not contain μ. More generally, for the 1000 simulations μ is contained in 94.9% of the confidence intervals. Also 1000 realisations of $\overline{X}_{40}$ are simulated for the considered minute. Figure 17(b) shows that, for $n = 40$, only one of the first fifty intervals does not contain μ. Note that the intervals are much smaller, because of the smaller values of $\overline{X}_40$ as explained above. For the 1000 simulations with $n = 40$, μ is contained in 95.2% of the constructed intervals. The fact that both percentages are practically equal to 95% shows the accuracy of the approximation. For this reason, and for simplicity of notation, the approximate 95% confidence interval of μ is assumed to be a true 95% confidence interval in the remainder of this thesis. In other words, instead of the approximate equation (14), it is assumed that

$$P\left(\mu - 1.96\sigma_{\overline{X}_n} \leq \overline{X}_n \leq \mu + 1.96\sigma_{\overline{X}_n}\right) = 0.95. \tag{15}$$

More generally, it is assumed in the remainder of this thesis that $\overline{X}_n$ is normally distributed with mean μ and variance $\sigma^2_{\overline{X}_n}$. From this assumption it follows that, for $\beta_1, \beta_2 \geq 0$,

$$P\left(\mu - \beta_1 \sigma_{\overline{X}_n} \leq \overline{X}_n \leq \mu + \beta_2 \sigma_{\overline{X}_n}\right) = P\left(-\beta_1 \leq \frac{\overline{X}_n - \mu}{\hat{\sigma}_{\overline{X}_n}} \leq \beta_2\right) = \Phi_{0,1}(\beta_2) - \Phi_{0,1}(-\beta_1), \quad (16)$$

with $\Phi_{0,1}$ the cumulative distribution function of the standard normal distribution. In the next chapter, equations (15) and (16) will be used to determine the minimum percentage of vehicles equipped with a uCAN that is necessary to accurately estimate the traffic speed.

6 Determining the necessary minimum percentage of vehicles equipped with a uCAN

In Chapter 5 it is already concluded from equation (12) that the more vehicles are equipped with a uCAN, the better the traffic speed can be estimated based on these vehicles. However, the less vehicles with a uCAN that are needed to estimate the traffic speed accurately, the sooner enough vehicles are equipped, so the sooner this new method for gathering speeds of vehicles can be used for traffic speed estimation. This means that then inductive loop detectors no longer are needed to estimate the traffic speed. Rijkswaterstaat therefore wants to minimize the number of vehicles equipped with a uCAN. On the other hand, n should be big enough for an accurate estimator of the traffic speed. Therefore, the optimal value of n is the minimum number of vehicles equipped with a uCAN so that the estimator $\overline{X}_n$ is a sufficiently accurate estimator. To determine this minimum, the meaning of sufficiently accurate should be defined, which is done with an accuracy requirement of NDW.

Real time traffic data that is collected by NDW is used to improve the utilisation of existing roads. NDW has set specific requirements for this collected data to be accepted in the database, see [32]. One of these requirements is that at least 95% of the collected traffic speeds needs to have a relative uncertainty of 5% or less. If this requirement is satisfied, this means that when the real traffic speed in 100 different minutes is 80 km/h, it is expected that 95% or more of the collected traffic speeds lie in the interval $[76, 84]$ km/h. Since the collected traffic speeds in this thesis are constructed with uCAN data, the requirement has to hold for the traffic speed estimated with n vehicles with a uCAN. In the notation of the previous section, NDW thus requires that

$$P\left(0.95\mu \leq \overline{X}_n \leq 1.05\mu\right) \geq 0.95. \tag{17}$$

So an estimator X_n is sufficiently accurate if it satisfies the NDW requirement (17). The question that then arises is what the minimum percentage of vehicles with a uCAN that is needed for a sufficiently accurate estimator of the traffic speed is.

The necessary minimum percentage of vehicles with a uCAN for which inequality (17) is satisfied, will be determined by making use of equation (15). Since equation (15) is expressed in terms of the speed mean μ and the speed standard deviation σ, these quantities need to be known. However, since only the speeds of vehicles with a uCAN are known and since not all vehicles are equipped with a uCAN nowadays, both quantities are not known. In this chapter, two methods will be developed to obtain information about these parameters, so that the necessary minimum percentage of vehicles with a uCAN can then be determined. The first method, discussed in Section 6.1, is based on historical values, while in Section 6.2 real-time information will be used.

6.1 Historical values used to determine the necessary minimum percentage

As mentioned above, the speed mean and the speed standard deviation are not known because not all vehicles are equipped with a uCAN. It is possible, however, to obtain information about μ and σ from historical values of these parameters, which can be derived from historical data. These historical values are used in this section to determine the necessary minimum percentage of vehicles equipped with a uCAN. In Section 6.1.1 a fixed minute is considered to determine this percentage, so N is known. However, when in the future in practice the traffic speed is estimated with speeds of vehicles with a uCAN, the total number of vehicles in this minute is not known, since there is only data from vehicles with a uCAN and not data from inductive loop detectors. It is therefore only known that the total number of vehicles is at least the observed number of vehicles with a uCAN. Therefore, it is assumed in Section 6.2.3 that the total number of vehicles in the considered minute is not known, and with this assumption the necessary minimum percentage of vehicles with a uCAN will also be determined.

6.1.1 Total number of vehicles known

From historical data, information about μ and σ can be obtained and used to determine the necessary minimum percentage of vehicles equipped with a uCAN. When the historical values of these parameters are used, and with the assumption of this section that in the considered minute the value of N is known, an answer to the following question can be given:

What is the minimum percentage of vehicles equipped with a uCAN that is necessary to accurately estimate the traffic speed in a fixed minute with a total number of N vehicles, when values of the speed mean and speed standard deviation of that minute can be seen as a random draw from historical values of the speed mean and the speed standard deviation?

Since $\overline{X}_n$ is sufficiently accurate if it satisfies inequality (17), this accuracy requirement is used to answer the question of this section. With the assumption that $\overline{X}_n$ is normally distributed with parameters μ and $\sigma^2_{\overline{X}_n}$ (as is approximately the case, see Section 5.2), the accuracy requirement can be rewritten in terms of μ and σ. By relating the two intervals of $\overline{X}_n$ obtained from equation (15) and inequality (17), it can be concluded that the NDW requirement is satisfied if

$$[0.95\mu, 1.05\mu] \supseteq \left[\mu - 1.96\sigma_{\overline{X}_n}, \mu + 1.96\sigma_{\overline{X}_n}\right], \tag{18}$$

since then

$$P\left(0.95\mu \leq \overline{X}_n \leq 1.05\mu\right) \geq P\left(\mu - 1.96\sigma_{\overline{X}_n} \leq \overline{X}_n \leq \mu + 1.96\sigma_{\overline{X}_n}\right) = 0.95.$$

Inclusion (18) above holds if

$$\sigma_{\overline{X}_n} \leq \frac{0.05}{1.96}\mu. \tag{19}$$

So the estimator of the traffic speed is sufficiently accurate if its standard deviation is smaller than this upper bound. The answer to the question of this section is obtained by solving this inequality for $p_{N,n} = \frac{n}{N}$, where $p_{N,n}$ is the percentage of vehicles equipped with a uCAN. Before this is done, another quantity will be introduced and used to give the answer. As can be seen, the above inequality is expressed in terms of the speed mean and the speed standard deviation. It is known that the standard deviation quantifies the variability of the data from the mean. An alternative, relative measure of variability in nonnegative data is the coefficient of variation. This coefficient of variation is defined as the ratio of the standard deviation to the mean, so $c_v = \frac{\sigma}{\mu}$. With this equation, inequality (19) can be expressed in terms of c_v. A lower bound, determined by c_v and N, for $p_{N,n}$ can then be found as follows:

$$\begin{aligned}
\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} &\leq \frac{0.05}{1.96}\mu \\
\frac{1}{n}\frac{N-n}{N-1} &\leq \left(\frac{0.05}{1.96}\right)^2 \frac{1}{c_v^2} \\
\frac{N}{n} - 1 &\leq \left(\frac{0.05}{1.96}\right)^2 \frac{1}{c_v^2}(N-1) \\
p_{N,n} &\geq \frac{1}{\left(\frac{0.05}{1.96}\right)^2 \frac{1}{c_v^2}(N-1) + 1}.
\end{aligned} \tag{20}$$

Note that c_v is always bigger than zero, so dividing by this quantity is allowed. As can be seen from the above inequality, the limit of the lower bound, as $c_v \to \infty$, is equal to 1. So if the coefficient of variation increases, the minimum percentage of vehicles with a uCAN also increases. This can be explained by the fact that if the coefficient of variation increases, either the speed standard deviation has increased or the speed mean has decreased. In the first case, the speeds of the vehicles differ more from each other, so more speeds are necessary to estimate the traffic speed accurately. In the second case, the traffic speed is lower, which implies a smaller interval required by the NDW and, thus, more vehicles with a uCAN are needed to satisfy the requirement of the NDW. Furthermore, the limit of the lower bound, as $N \to \infty$, is equal to 0. This means that if the total number of vehicles in one minute increases, the minimum percentage of vehicles decreases to zero.

From inequality (20) it follows that, since N is known in this section, the minimum percentage of vehicles with a uCAN is determined by the value of c_v. As an example, the minute from Section 5.2 is considered, where a total of 53 vehicles pass a certain point, so $N = 53$. This leads to Figure 18, where the minimum percentage is shown as a function of c_v. Since, as mentioned above, only a total number of vehicles in one minute between $N = 30$ and $N = 80$ is considered, the functions for these boundary values are also shown in the figure.

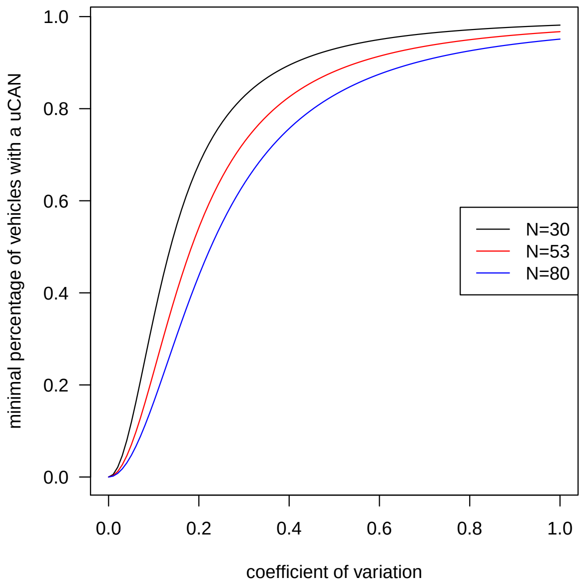


Figure 18: Minimum percentage of vehicles with a uCAN determined by c_v

As can be seen from Figure 18, a minor change in the value of a relatively small c_v has a large influence on the minimum percentage, while for relatively big values of c_v this influence is much smaller. This result can be explained: for a constant μ, a small c_v corresponds to a small σ, so a minor change in c_v means a relatively large change in the variation of the speeds of the vehicles. This leads to a relatively large increase or decrease of the necessary minimum percentage of vehicles equipped with a uCAN. An opposite reasoning holds for large values of c_v. As already concluded from inequality (20), it can also be seen from the figure that the minimum percentage approaches 1 as $c_v \to \infty$, and that for bigger values of N the minimum percentage decreases.

The figure above shows that, when the value of c_v of the considered minute is known, the minimum percentage of vehicles with a uCAN in that minute can be determined. However, the value of c_v of a specific minute is not known, only historical values are known. Therefore c_v is considered as a random variable, which is randomly drawn from historical values of c_v. Let $c_{v,1}, \ldots, c_{v,6840}$ be the sorted historical coefficients of variation of all minutes of six days, derived from the corresponding historical values of the speed mean μ_i and the speed standard deviation σ_i. To see if the value of c_v depends on the value of N, a scatterplot of the historical values of c_v against the total number of vehicles N of the corresponding minute is shown in Figure 19.

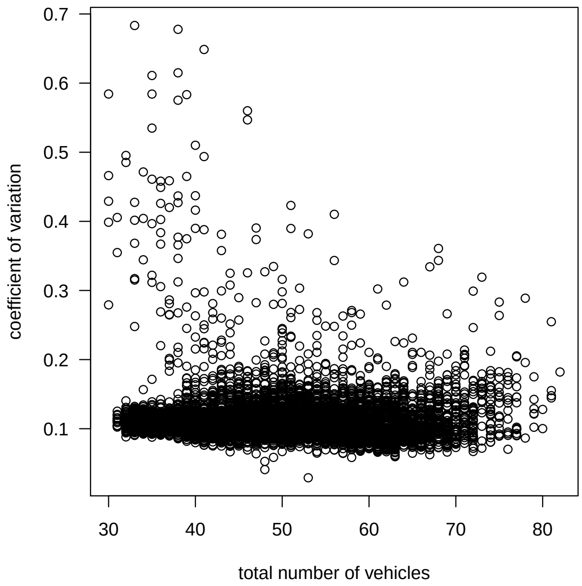


Figure 19: Scatterplot of historical values of c_v against N

As can be seen from Figure 19, the minimum total number of vehicles is equal to 30. This is a result of the chosen intensity in Fosim, described in Section 3.3. Furthermore, the maximum total number of vehicles is equal to 82, due to the capacity of the two-lane highway with one entrance ramp. However, there are few minutes where the total number of vehicle is close to the maximum total number.

From Figure 19 it follows that the historical values of c_v take on about the same values for different values of the total number of vehicles in one minute. To see if the value of c_v is indeed the same for different values of the total number of vehicles, a permutation test is used. First, linear regression is performed on the observed historical pairs of c_v and N, to obtain the straight line through these points that minimizes the sum of the squared residuals. The slope of this straight line, denoted by β, is the test statistic for the permutation test. The null hypothesis that the value of c_v is equal for all values of N can then be states as the hypothesis that $\beta = 0$. The observed value of the test statistic, which is the slope of the mentioned straight line for the observed historical values, is equal to $-2.592 \cdot 10^{-5}$.

Second, the historical pairs of c_v and N are rearrangement, i.e. to each historical c_v another N is assigned by permuting the historical values. For these permuted pairs, a different straight line through these points is obtained via linear regression. The slope of this straight line is different than the observed value of the test statistic. By permuting the data pairs 1000 times and determining the slope of the straight line through each of these permuted data pairs, 1000 different values of the test statistic are obtained. The null hypothesis can then be tested by determining the percentage of the values for which the difference of that value and zero, i.e.

the absolute value, is bigger than the absolute value of the observed value of the test statistic. In Figure 20, the 1000 values of the test statistic are shown in a histogram, together with the observed value of the test statistic.

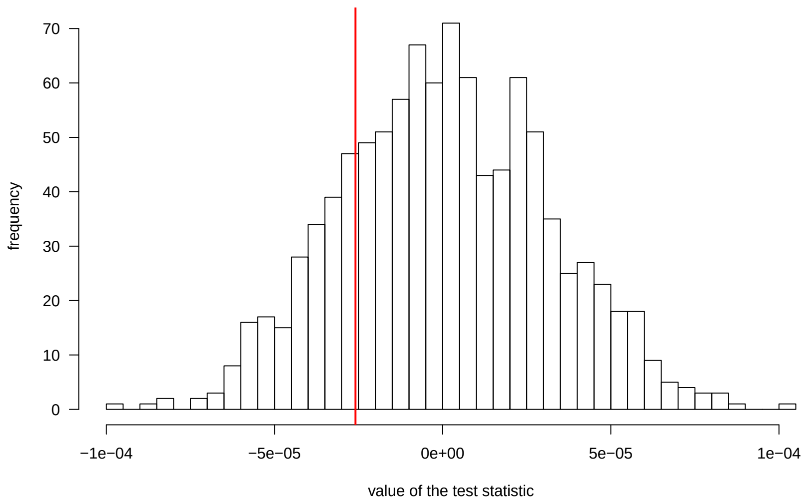


Figure 20: Histogram of the values of the test statistic

As can be seen from the figure, the observed value of the test statistic is not a strange value compared to the other obtained values. The percentage of the values of which the absolute value is bigger than the observed value of the test statistic is equal to 58.9%. Since this value is bigger than a significance level of 5%, or even 10%, the null hypothesis is not rejected. It can therefore be assumed that the value of c_v is the same for different values of N.

As a consequence, it can also be assumed that the value of c_v of the considered minute is a random draw from all historical values, which means that $P(c_v = c_{v,i}) = \frac{1}{8640}$ for $i = 1, \ldots, 8640$. From this distribution, an a priori fixed value of the coefficient of variation will be deduced, which will be used to obtain an answer to the question of this section.

A first idea for this a priori fixed value is the 90% quantile of c_v. This quantile is equal to $c_{v,7776}$, since the historical values are sorted and thus $P(c_v \leq c_{v,7776}) = 0.90$. Based on this fixed value for c_v, and the total number of vehicles in the considered minute, the necessary minimum percentage of vehicles equipped with a uCAN for that minute can be determined by (20). For this choice of the a priori fixed value, 90% of the minutes will have a value of c_v that is smaller than or equal to $c_{v,7776}$. As a result, the necessary minimum number of vehicles with a uCAN determined with the 90% quantile of c_v will be higher than actually necessary in 90% of the minutes. For these minutes, the probability that the estimator is contained in the corresponding interval $[0.95\mu, 1.05\mu]$ is therefore higher than the required 0.95 by NDW. As a consequence, the estimator will be contained in this interval in more than 95% of the minutes. Although the accuracy requirement of NDW will thus be satisfied, the minimum percentage of vehicles with a uCAN, and thus the a priori fixed value of c_v, could be chosen smaller, while the NDW requirement will still be satisfied.

A better choice for the a priori fixed value may therefore be the 50% quantile, also called the median. When this value is chosen, 50% of the values of c_v will be smaller than the a priori fixed value, which means that, in 50% of the minutes, the actual necessary minimum percentage of vehicles with a uCAN will be smaller than the necessary minimum percentage determined with the median, and bigger in the other 50% of the minutes. This result is better than the previous choice, but it has a different disadvantage. When the differences between the actual necessary minimum of the 50% of the minutes with a c_v smaller than the median and the necessary minimum computed with the median is on average smaller than the same difference for the minutes with a c_v bigger than the median, the resulting percentage of estimators that is contained in the corresponding interval will be bigger, and vice versa. To ensure that these differences compensate each other, the expected value instead of the median of c_v is chosen as a priori fixed value of c_v. For this value, the probability that the estimator is contained in the corresponding interval will on average be equal to 0.95, so the percentage of minutes for which $0.95\mu \leq \overline{X}_n \leq 1.05\mu$ will be equal to 95%.

Since the distribution of c_v is known, this expected value can be computed:

$$E\left[c_v\right] = \sum_{i=1}^{8640} c_{v,i} P(c_v = c_{v,i}) = \frac{1}{8640} \sum_{i=1}^{8640} c_{v,i},$$

which is the mean of the historical values. The a priori fixed value of c_v is therefore equal to the mean of the historical values. Consider the figure below for a histogram of the historical values of c_v, where the vertical red line represents the average value of the historical values.

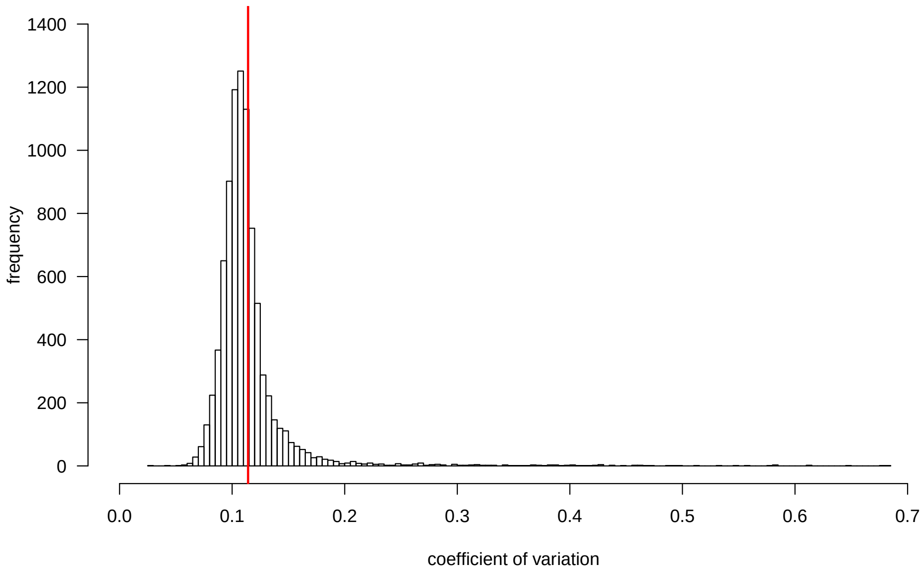


Figure 21: Histogram of historical values of c_v

The average value of the historical values of c_v is equal to 0.1142, which is the a priori fixed value that will be used. By plugging this a priori value of c_v into the inequality (20) of $p_{N,n}$, the necessary minimum percentage of vehicles equipped with a uCAN is determined for each value of N. The figure below shows this minimum percentage as a function of N.

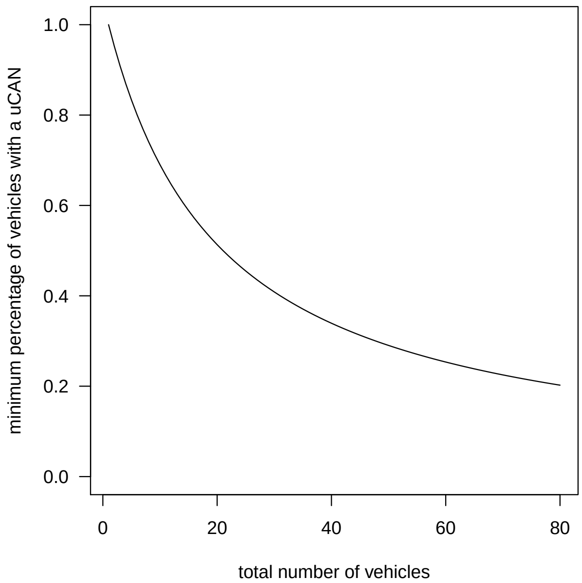


Figure 22: Necessary minimum percentage of vehicles with a uCAN with a priori fixed $c_v = 0.1142$

It can be seen from Figure 22 that a bigger value of N leads to a smaller value of the necessary minimum percentage of vehicles with a uCAN, which means that relatively less vehicles with a uCAN are necessary when the total number of vehicles is bigger. The figure also shows that if N is known, and the a priori fixed value of c_v, the necessary minimum number can be determined. The answer to the question of this section can then be given:

The minimum percentage of vehicles equipped with a uCAN that is necessary to accurately estimate the traffic speed is equal to

$$\frac{1}{\left(\frac{0.05}{1.96}\right)^2 \frac{1}{c_v^2}(N-1)+1},$$

with N the known total number of vehicles in the considered minute and the a priori fixed value $c_v = 0.1142$, where c_v is considered as a random draw from historical values of the coefficient of variation.

For the example above, where $N = 53$, it follows that at least 27.8% of the vehicles need to be equipped with a uCAN. This means that if 15 vehicles or more in the considered minute were equipped with a uCAN, then the estimator of the traffic speed is sufficiently accurate. In this section, the total number of vehicles is known. Since in practice, this number is not known, in the next section it is assumed that N is not known.

6.1.2 Total number of vehicles not known

In the previous section, an a priori fixed value of c_v and the known total number of vehicles in the considered minute are used to determine the necessary minimum percentage of vehicles with a uCAN by inequality (20). However, only determining an a priori fixed value of c_v does not yield a solution that can be used in practice, since N is then not known. In this section it is therefore assumed that N is not known and an answer to the following question will be given:

What is the minimum percentage of vehicles equipped with a uCAN that is necessary to accurately estimate the traffic speed, when the values of N and c_v can be seen as random draws from the historical values of N and c_v respectively?

Also in this section inequality (20) is used to answer the question above. In contrast to the previous section, not only c_v, but also N is not known. Both these quantities are therefore considered as random variables. Then also the corresponding minimum percentage M is a random variable, defined by

$$M = \frac{1}{\left(\frac{0.05}{1.96}\right)^2 \frac{1}{c_v^2}(N-1)+1}.$$

Similar to the previous section, it is assumed that M is a random draw from historical values. More specific, M is randomly drawn from the historical minimum percentages $m_1, \ldots, m_{8640}$, which can be computed based on the corresponding historical values of the coefficient of variation and the total number of vehicles in the specific minute. Let $(N_1, c_{v,1}), \ldots, (N_{8640}, c_{v,8640})$ be the historical values of 8640 minutes. For each of the pairs $(N_i, c_{v,i})$ the corresponding necessary minimum percentage m_i can then be computed by

$$m_i = \frac{1}{\left(\frac{0.05}{1.96}\right)^2 \frac{1}{c_{v,i}^2}(N_i-1)+1} \qquad \text{for } i = 1, \ldots, 8640.$$

The necessary minimum percentage of vehicles that need to be equipped with a uCAN is determined by these values. Similar to the previous section, the 90% quantile of these numbers results in a necessary minimum percentage that will be too high, since then the percentage of minutes for which the traffic speed is accurately estimated will be bigger than 95%. When the expected value of M is chosen as the necessary minimum percentage, the minutes for which the actual minimum percentage is less than this necessary minimum percentage compensate the minutes for which the actual minimum percentage is bigger. The estimator of the traffic speed will be more accurate in the former minutes, while less accurate in the latter minutes. By choosing the mean of M as necessary minimum percentage, the average percentage of minutes for which the traffic speed is accurately estimated will be 95%.

Since the distribution of M is known, its expected value can be calculated:

$$E[M] = \sum_{i=1}^{8640} m_i P(M = m_i) = \frac{1}{8640} \sum_{i=1}^{8640} m_i.$$

The necessary minimum percentage of vehicles with a uCAN is therefore equal to the average percentage of the observed historical values, which is equal to 30.6%. In the histogram below, the historical values of the minimum percentages are shown together with the mean of these minimum percentages.

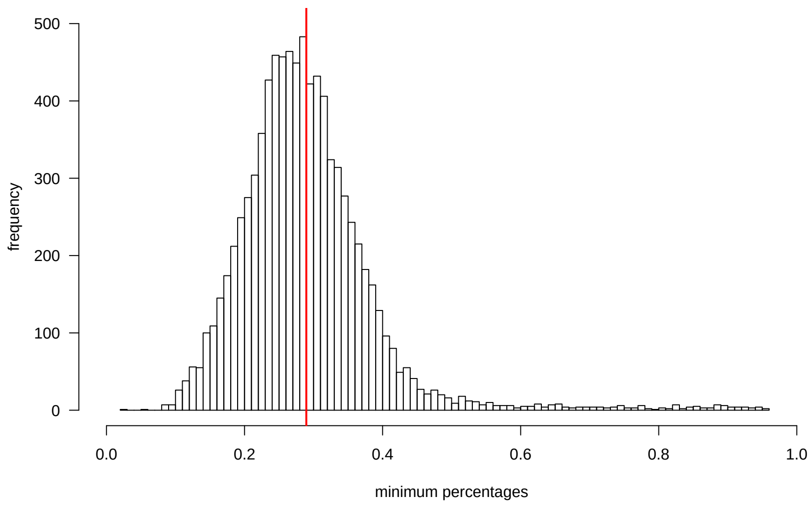


Figure 23: Historical minimum percentages of vehicles with a uCAN

The answer to the question of this section can then be given:

The minimum percentage of vehicles equipped with a uCAN that is necessary to accurately estimate the traffic speed, where the values of N and c_v are considered as random draws from the historical values of N and c_v respectively, is equal to 30.6%*.*

The necessary minimum percentage obtained in the example of Section 6.1.1 is lower than the percentage obtained in this section. This means that the necessary minimum percentage obtained in this section will therefore lead to an accurate estimator for the traffic speed for the considered minute in the example.

However, for large values of N, the necessary absolute number of vehicles with a uCAN corresponding to this percentage is large compared to small values of N. It is therefore possible that the estimator based on 30.6% vehicles with a uCAN is more accurate for larger values of N. For example, the estimated traffic speed of 60 vehicles based on 19 vehicles is possibly more accurate than the estimated traffic speed of 30 vehicles based on only 10 vehicles, since the variance in the estimated traffic speeds based on 10 vehicles is bigger than the variance in the estimated traffic speeds based on 19 vehicles. To investigate this, the obtained minimum percentage will be used in inequality (20) to construct the following upper bound for c_v as a function of N:

$$c_v \leq \sqrt{\frac{\left(\frac{0.05}{1.96}\right)^2 (N-1)}{\frac{1}{0.306} - 1}}.$$

This upper bound is shown in the scatterplot of the combinations $(N_i, c_{v,i})$ in the figure below.

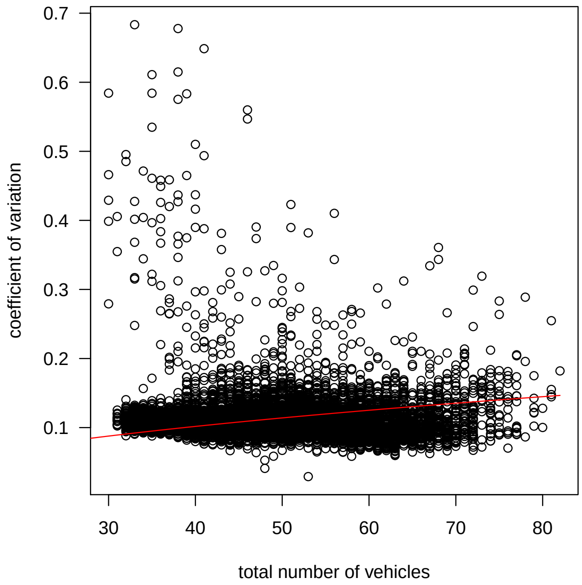


Figure 24: Scatterplot of historical values of c_v against N

The red line shown in Figure 24 represents the upper bound for c_v. This upper bound is constructed with the average value of the corresponding minimum percentages of the historical values. As can be clearly seen from the figure, for large values of N relatively more points are below the red line compared to small values of N, which means that for these values of N the estimator is more often accurate than for small values. For different intervals of N the percentage of points below the upper bound is shown in Table 3.

Interval of N	$N \leq 45$	$45 < N \leq 60$	$N > 60$
Percentage	37.0%	70.1%	82.4%

Table 3: Percentages of points below upper bound for different intervals of N.

As can be seen from the table, the percentages differ much between the different intervals of N. Therefore, when 30.6% of the vehicles are equipped with a uCAN, the estimator $\overline{X}_n$ is less often than desired sufficiently accurate when the total number of vehicles is 45 or less, while the estimator is more often sufficiently accurate when the total number is 60 or more.

This result suggests to consider a necessary minimum number of vehicles with a uCAN instead of a minimum percentage. For this, the question of this section is slightly modified:

What is the minimum ***number*** *of vehicles equipped with a uCAN that is necessary to accurately estimate the traffic speed, when the values of N and c_v can be seen as random draws from the historical values of N and c_v respectively?*

Instead of considering M, the random variable NM, which corresponds to the minimum number, will be considered. Similar to before, the necessary minimum number of vehicles with a uCAN is then equal to the expected value of NM. Similar to the expected value of M, this expected value is equal to the average value of the historical values $N_i m_i$. This average value is equal to 13.99, so $n = 14$ vehicles have to be equipped with a uCAN. To see if this answer results in percentages of sufficiently accurate estimators that differ less for different values of N, the following upper bound for c_v is constructed:

$$c_v \leq \sqrt{\frac{\left(\frac{0.05}{1.96}\right)^2 (N-1)}{\frac{N}{n}-1}}, \qquad \text{with } n = 14.$$

This upper bound is shown as the red line in Figure 25, together with the scatterplot of the historical values of N and c_v.

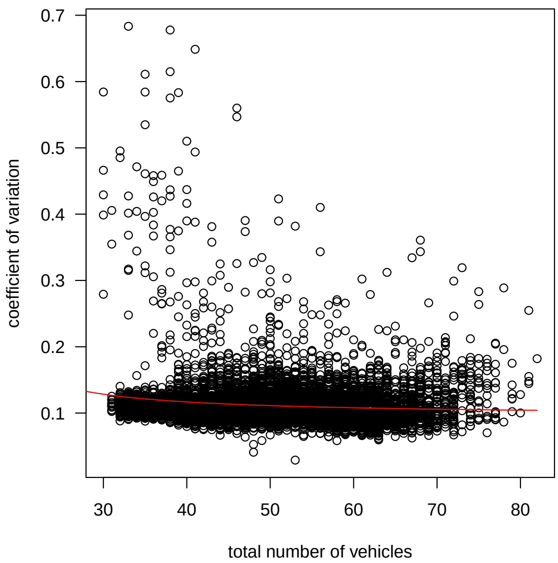


Figure 25: Scatterplot of historical values of c_v against N

As can be seen from this figure, the amount of points below the red line differs less for different values of N. The percentages of points below the upper bound are shown in Table 4 for the same intervals for N as in Table 3.

Interval of N	$N \leq 45$	$45 < N \leq 60$	$N > 60$
Percentage	74.4%	59.1%	54.3%

Table 4: Percentages of points below upper bound for different intervals of N.

These percentages are closer to each other compared to the percentages in Table 3, so the total number of vehicles has less influence on the percentage of minutes for which the estimator is sufficiently accurate. With the obtained value of n, an answer can be given to the question of this section, where the minimum percentage is replaced by the minimum number of vehicles with a uCAN:

*The minimum **number** of vehicles equipped with a uCAN that is necessary to accurately estimate the traffic speed, when the values of N and c_v are considered as random draws from the historical values of N and c_v respectively, is equal to 14.*

Note that in this section, the analysis is based on historical values. Apart from Section 6.1.1, where it was assumed that N is known, no information of the minute itself is used to obtain the answers to the questions. In the next section, real-time information obtained from uCAN data of the considered minute will be used to determine the minimum percentage of vehicles with a uCAN necessary to accurately estimate the traffic speed.

6.2 Real-time information used to determine the necessary minimum percentage

In the previous section, the a priori fixed value of c_v, based on historical values, is used to determine the necessary minimum percentage of vehicles equipped with a uCAN. However, this value does not contain real-time information. An alternative approach is to use real-time information, since historical data might not be representative for the considered minute. Moreover, in the computations of the traffic speed nowadays, with data from inductive loop detectors, no historical data is used, only real-time information. The real-time information that is considered in this thesis is obtained from vehicles with a uCAN. In this section, instead of historical values, the available uCAN speeds are used to determine the necessary minimum percentage of vehicles with a uCAN. Again, two different situations are considered, namely the situation that N is known and the situation that N is not known. Sections 6.2.1 and 6.2.2 will describe two different methods to determine the minimum percentage, where in both sections it is assumed that the total number of vehicles in the considered minute is known. In Section 6.2.3 the situation where N is not known will be discussed.

6.2.1 Total number of vehicles known, method 1

Compared to the questions of the previous section, real-time information is used in this section to answer the following question:

What is the minimum percentage of vehicles equipped with a uCAN that is necessary to accurately estimate the traffic speed in a fixed minute with a total number of N vehicles, when the analysis is based on real-time information obtained from uCAN data?

When real-time information is used, the speed mean μ and the speed standard deviation σ are still not known, since only the speeds of the n vehicles with a uCAN are known. This means that $X_1, \ldots, X_n$ are known, and thus $\overline{X}_n$ and the variance $S^2_{n,N}$ of these speeds as well, where

$$S^2_{n,N} = \frac{1}{n} \sum_{i=1}^{n} \left(X_i - \overline{X}_n\right)^2 . \tag{21}$$

These quantities can be used to determine the necessary minimum percentage of vehicles with a uCAN to accurately estimate the traffic speed. To determine this percentage, the accuracy of NDW is still required and thus inequality (17) has to hold. Since $\overline{X}_n$ is known, the requirement of NDW is written as the equivalent requirement:

$$P\left(\frac{1}{1.05}\overline{X}_n \leq \mu \leq \frac{1}{0.95}\overline{X}_n\right) \geq 0.95. \tag{22}$$

Furthermore, from (13) it is known that $\overline{X}_n$ is approximately normally distributed with the given parameters μ and $\sigma^2_{\overline{X}_n}$, resulting in (15). Since the rewritten NDW requirement is given in known bounds for μ, equation (15) is rewritten as well:

$$P\left(\overline{X}_n - 1.96\sigma_{\overline{X}_n} \leq \mu \leq \overline{X}_n + 1.96\sigma_{\overline{X}_n}\right) = 0.95. \tag{23}$$

By comparing the bounds for μ, the necessary minimum percentage can be determined. However, $\sigma_{\overline{X}_n}$ is not known since σ is not known. During the installation of the uCAN into vehicles, more and more vehicles can provide their speeds to the traffic data collection system for calculations. With this real-time information the speed standard deviation can be estimated so that an estimator of the standard deviation of the sample mean, denoted by $\hat{\sigma}_{\overline{X}_n}$, can be obtained. To decide how many vehicles need to be equipped with a uCAN to accurately estimate the traffic speed, $\sigma_{\overline{X}_n}$ has to be estimated. Similar to [30], this is done by constructing an unbiased estimator $\hat{\sigma}$ for the speed standard deviation.

A natural estimator for the speed variance σ^2 is the variance of the speeds of the vehicles with a uCAN (21) and its expected value can be computed.

$$\begin{aligned}
E\left[S_{n,N}^2\right] &= E\left[\frac{1}{n}\sum_{i=1}^{n}\left(X_i - \overline{X}_n\right)^2\right] = E\left[\frac{1}{n}\sum_{i=1}^{n}X_i^2 - 2\overline{X}_n\frac{1}{n}\sum_{i=1}^{n}X_i + \overline{X}_n^2\right] \\
&= \frac{1}{n}\sum_{i=1}^{n}E\left[X_i^2\right] - 2E\left[\overline{X}_n^2\right] + E\left[\overline{X}_n^2\right] = E\left[X_1^2\right] - E\left[\overline{X}_n^2\right] \\
&= \mathrm{Var}\left(X_1\right) + E\left[X_1\right]^2 - \mathrm{Var}\left(\overline{X}_n\right) - E\left[\overline{X}_n\right]^2 = \sigma^2 + \mu^2 - \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right) - \mu^2 \\
&= \sigma^2\left(1 - \frac{1}{n}\left(\frac{N-n}{N-1}\right)\right) = \sigma^2\,\frac{N\left(n-1\right)}{n\left(N-1\right)}.
\end{aligned}$$

Hence, $S_{n,N}^2$ is a biased estimator of σ^2. By dividing $S_{n,N}^2$ with the factor $\frac{N(n-1)}{n(N-1)}$, the unbiased estimator of the speed variance, denoted by $\hat{\sigma}^2$ is obtained:

$$\hat{\sigma}^2 = \frac{N-1}{N\left(n-1\right)}\sum_{i=1}^{n}\left(X_i - \overline{X}_n\right)^2.$$

An unbiased estimator of the variance of the sample mean is obtained by plugging the unbiased estimator $\hat{\sigma}^2$ into the expression for $\sigma_{\overline{X}_n}^2$, given in (12):

$$\hat{\sigma}_{\overline{X}_n}^2 = \frac{\hat{\sigma}^2}{n}\left(\frac{N-n}{N-1}\right).$$

In the previous section, $\sigma_{\overline{X}_n}^2$ is a fixed value for each n, while in this section the variance of the sample mean has to be estimated with the above formula. This estimate depends on $\hat{\sigma}$ and thus on the speeds of the n vehicles with a uCAN. Because there are different possible samples of size n from the total number of N vehicles, different speeds of vehicles with a uCAN are used to estimate the speed variance. So for a specific value of n different estimators of $\sigma_{\overline{X}_n}^2$ can be obtained.

By combining the central limit theorem for finite populations and Slutsky's theorem [33] it is proved in [34] that the result obtained from (13) also holds when $\sigma_{\overline{X}_n}$ is replaced by the unbiased estimator $\hat{\sigma}_{\overline{X}_n}$. Equation (23) is therefore true with the estimator for $\sigma_{\overline{X}_n}$ plugged in as well, i.e.

$$P\left(\overline{X}_n - 1.96\hat{\sigma}_{\overline{X}_n} \le \mu \le \overline{X}_n + 1.96\hat{\sigma}_{\overline{X}_n}\right) = 0.95.$$

Then both bounds in that equation are known, so these bounds can be compared to the bounds of (22). Similar to the derivation of inequality (19), it can be concluded that for

$$\hat{\sigma}_{\overline{X}_n} \le \min\left\{\frac{\frac{1}{0.95} - 1}{1.96}\overline{X}_n, \frac{1 - \frac{1}{1.05}}{1.96}\overline{X}_n\right\} = \frac{1 - \frac{1}{1.05}}{1.96}\overline{X}_n \tag{24}$$

the NDW requirement is satisfied, since then

$$\begin{aligned}
\overline{X}_n - 1.96\hat{\sigma}_{\overline{X}_n} &\ge \overline{X}_n - 1.96\frac{1 - \frac{1}{1.05}}{1.96}\overline{X}_n = \frac{1}{1.05}\overline{X}_n \\
\overline{X}_n + 1.96\hat{\sigma}_{\overline{X}_n} &\le \overline{X}_n + 1.96\frac{1 - \frac{1}{1.05}}{1.96}\overline{X}_n = \frac{1.10}{1.05}\overline{X}_n < \frac{1}{0.95}\overline{X}_n,
\end{aligned} \tag{25}$$

and thus

$$P\left(\frac{1}{1.05}\overline{X}_n \leq \mu \leq \frac{1}{0.95}\overline{X}_n\right) \geq P\left(\overline{X}_n - 1.96\hat{\sigma}_{\overline{X}_n} \leq \mu \leq \overline{X}_n + 1.96\hat{\sigma}_{\overline{X}_n}\right) = 0.95.$$

Inequality (24) can be rewritten as

$$\frac{\hat{\sigma}_{\overline{X}_n}}{\overline{X}_n} \leq \frac{1-\frac{1}{1.05}}{1.96}. \tag{26}$$

For a fixed minute with a total number of N vehicles, 1000 times n vehicles can be drawn randomly from the N vehicles in total. For each n, 1000 realisations of the sample mean and the standard deviation of the sample mean, denoted by $\overline{X}_{n,i}$ and $\hat{\sigma}_{\overline{X}_{n,i}}$ for $i = 1, \ldots, 1000$ respectively, can be computed, and thus 1000 values of the left hand side of inequality (26) as well. Analogous to the methods of the previous section, it is desired that the mean of these 1000 values satisfies inequality (26), since then the percentage of estimators that lies in the interval of NDW will on average be 95%. The necessary minimum percentage of vehicles equipped with a uCAN is therefore equal to the minimum percentage $\frac{n}{N}$ for which the mean of the 1000 realisations of the left hand side of inequality (26) is smaller than or equal to $\frac{1-\frac{1}{1.05}}{1.96}$. The answer to the question of this section can thus be given:

The minimum percentage of vehicles equipped with a uCAN that is necessary to accurately estimate the traffic speed is equal to the minimum percentage $\frac{n}{N}$, with N the known total number of vehicles in the considered minute, for which

$$\frac{1}{1000}\sum_{i=1}^{1000}\frac{\hat{\sigma}_{\overline{X}_{n,i}}}{\overline{X}_{n,i}} \leq \frac{1-\frac{1}{1.05}}{1.96}$$

where the analysis is based on real-time information obtained from uCAN data.

As an example, the minute in the previous section, with $N = 53$, is considered. For each n, the values of $\overline{X}_{n,i}$ and $\hat{\sigma}_{\overline{X}_{n,i}}$ are computed, and the latter divided by the former is plotted against the percentage of vehicles equipped with a uCAN in the figure below.

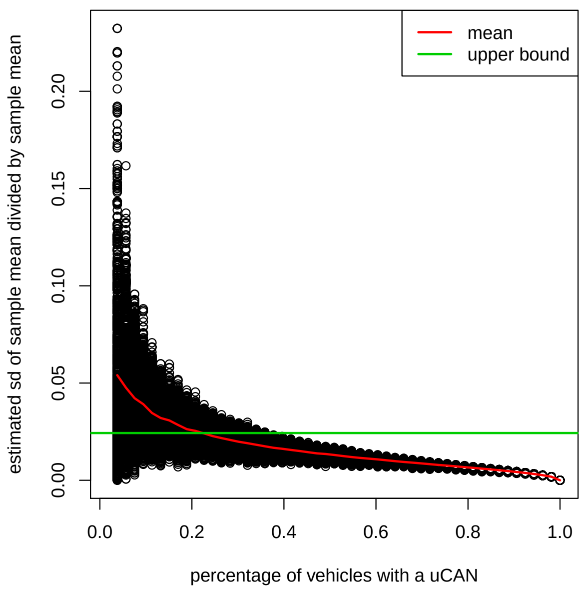


Figure 26: $\frac{\hat{\sigma}_{\overline{X}_n}}{\overline{X}_n}$ plotted against $\frac{n}{N}$

As can be seen from Figure 26, the values of $\hat{\sigma}_{\overline{X}_n}$ divided by $\overline{X}_n$ decrease to zero when the percentage of vehicles with a uCAN increases to 100%. This is a consequence of the fact that the value of $\hat{\sigma}_{\overline{X}_n}$ decreases when more vehicles are equipped with a uCAN, since the estimator of the traffic speed is then more accurate. It is desired that the mean of the realisations satisfies the requirement of NDW. Therefore, the the mean is calculated for each percentage of the vehicles with a uCAN and the result is shown in the figure by the red line. The necessary minimum percentage of vehicles with a uCAN is then equal to the minimum percentage for which this mean is smaller or equal to the upper bound of inequality (26), represented by the green line in Figure 26. For this minute, with $N = 53$, the necessary minimum percentage is equal to 23.7%, which means that if at least 13 vehicles were equipped with a uCAN, the estimator of the traffic speed is sufficiently accurate.

In contrast to Section 6.1.1, this result does not apply to all minutes with $N = 53$, since in this section real-time information about the minute is used. This information contains information about the value of c_v of the considered minute. For different minutes with $N = 53$ the value of c_v is not the same, so the resulting necessary minimum percentage will differ, since a bigger value of c_v leads to a bigger necessary minimum percentage of vehicles with a uCAN.

The derivation of the method of this section involved inequality (24), where actually this upper bound was taken too small for the upper bound of μ, which can be seen from the strict inequality (25). The requirement (26) that was used is too strict. As a result, the necessary minimum percentage obtained with this method might be higher than needed. In the next section a different method will therefore be derived to obtain the necessary minimum percentage of vehicles with a uCAN, where the normal approximation of the distribution of $\overline{X}_n$ will be used to approximate the probability of (22).

6.2.2 Total number of vehicles known, method 2

In this section, real-time information is used as well to give an answer to the question stated at the beginning of Section 6.2.1. However, for this answer a different approach will be used to determine the necessary minimum percentage of vehicles equipped with a uCAN.

From equation (16) and the above mentioned result that this equation also holds when $\sigma_{\overline{X}_n}$ is replaced by its estimator $\hat{\sigma}_{\overline{X}_n}$, it is known that

$$P\left(\mu - \beta_1 \hat{\sigma}_{\overline{X}_n} \leq \overline{X}_n \leq \mu + \beta_2 \hat{\sigma}_{\overline{X}_n}\right) = \Phi_{0,1}(\beta_2) - \Phi_{0,1}(-\beta_1), \tag{27}$$

with $\Phi_{0,1}$ the cumulative distribution function of the standard normal distribution. Moreover, it is known that it also holds for this probability that

$$P\left(\mu - \beta_1 \hat{\sigma}_{\overline{X}_n} \leq \overline{X}_n \leq \mu + \beta_2 \hat{\sigma}_{\overline{X}_n}\right) = P\left(\overline{X}_n - \beta_2 \hat{\sigma}_{\overline{X}_n} \leq \mu \leq \overline{X}_n + \beta_1 \hat{\sigma}_{\overline{X}_n}\right). \tag{28}$$

By choosing $\beta_1 = \frac{0.05}{0.95} \frac{\overline{X}_n}{\hat{\sigma}_{\overline{X}_n}}$ and $\beta_2 = \frac{0.05}{1.05} \frac{\overline{X}_n}{\hat{\sigma}_{\overline{X}_n}}$, the probability that μ lies in the interval required by NDW can be computed as follows:

$$P\left(\frac{1}{1.05}\overline{X}_n \leq \mu \leq \frac{1}{0.95}\overline{X}_n\right) = P(\overline{X}_n - \beta_2 \hat{\sigma}_{\overline{X}_n} \leq \mu \leq \overline{X}_n + \beta_1 \hat{\sigma}_{\overline{X}_n}) = \Phi_{0,1}(\beta_2) - \Phi_{0,1}(-\beta_1),$$

where the last equation follows from equations (27) and (28).

For a specific minute, with N known, let p_n denote the true probability that μ lies in the interval $\left[\frac{1}{1.05}\overline{X}_n, \frac{1}{0.95}\overline{X}_n\right]$. The accuracy requirement of NDW is then satisfied if $p_n \geq 0.95$. Since there are $\binom{N}{n}$ possible random samples of size n, the true probability can be computed as the mean probability of all these samples, i.e.

$$\begin{aligned} p_n &= \sum_{i=1}^{\binom{N}{n}} P\left(\frac{1}{1.05}\overline{X}_n \leq \mu \leq \frac{1}{0.95}\overline{X}_n \middle| i^{th} \text{ sample is drawn}\right) P\left(i^{th}\text{sample is drawn}\right) \\ &= \frac{1}{\binom{N}{n}} \sum_{i=1}^{\binom{N}{n}} p_{n,i}, \end{aligned}$$

where $p_{n,i}$ denotes the probability that μ is contained in the corresponding interval of the i^{th} sample. Let $\overline{X}_{n,i}$ and $\hat{\sigma}_{\overline{X}_{n,i}}$ be the sample mean and the standard deviation of the sample mean respectively of the i^{th} sample, then:

$$p_{n,i} = \Phi_{0,1}\left(\frac{0.05}{1.05}\frac{\overline{X}_{n,i}}{\hat{\sigma}_{\overline{X}_{n,i}}}\right) - \Phi_{0,1}\left(-\frac{0.05}{0.95}\frac{\overline{X}_{n,i}}{\hat{\sigma}_{\overline{X}_{n,i}}}\right).$$

However, for large values of N, for example $N = 53$, the total number of possible samples for most of the values of n is too large to consider all possible random samples. Therefore, $B = \min\left\{\binom{N}{n}, 1000\right\}$ identically distributed realisations $P_{n,1}, \ldots, P_{n,B}$ are drawn from the total number of random samples, where $P_{n,i} \in \{p_{n,1}, \ldots, p_{n,\binom{N}{n}}\}$ with $P(P_{n,1} = p_{n,j}) = \frac{1}{\binom{N}{n}}$ for $j = 1, \ldots, \binom{N}{n}$. Then p_n can be estimated by the average value $\overline{P}_n$ of these probabilities:

$$\overline{P}_n = \frac{1}{B}\sum_{i=1}^{B} P_{n,i}.$$

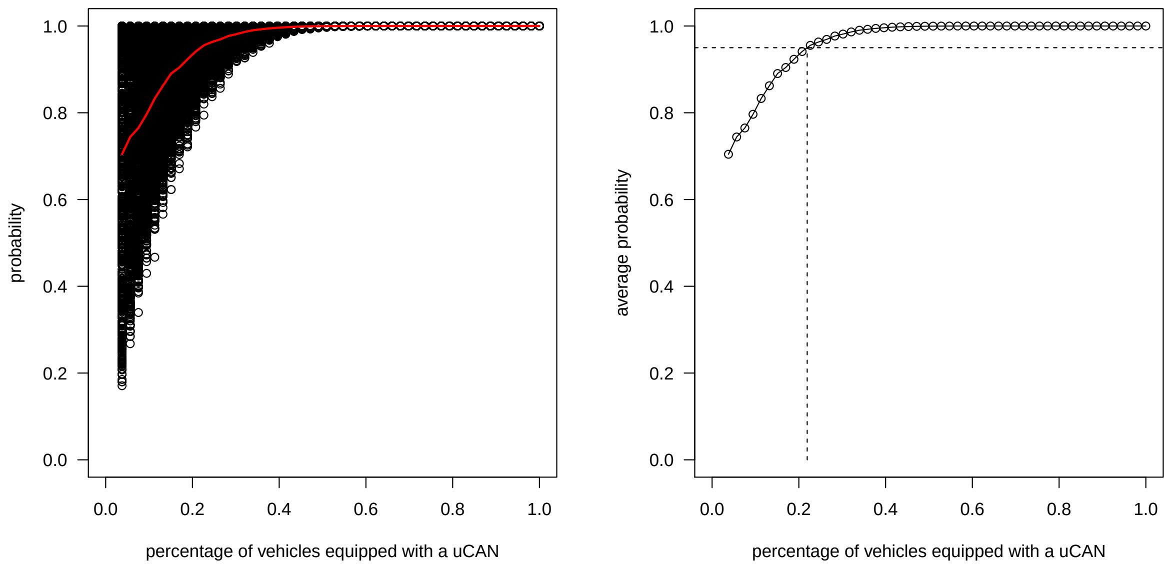
The expected value of $\overline{P}_n$ is equal to

$$E\left[\overline{P}_n\right] = \frac{1}{B}\sum_{i=1}^{B} E[P_{n,i}] = E[P_{n,1}] = \sum_{i=1}^{\binom{N}{n}} p_{n,i}P(P_n = p_{n,i}) = \frac{1}{\binom{N}{n}}\sum_{i=1}^{\binom{N}{n}} p_{n,i} = p_n,$$

so $\overline{P}_n$ is an unbiased estimator of the probability p_n. The requirement of NDW can then be stated in terms of the average probability: $\overline{P}_n \geq 0.95$. With this inequality, the answer based on the method of this section of the question of Section 6.2.1 can be given:

The minimum percentage of vehicles equipped with a uCAN that is necessary to accurately estimate the traffic speed, with N known, determined by using real-time information obtained from uCAN data, is equal to the minimum percentage $\frac{n}{N}$ *for which* $\overline{P}_n \geq 0.95$.

Consider the minute of the previous section, where it is known that 53 vehicles have passed a certain point in this minute. For this minute, B times n vehicles are drawn randomly from the N vehicles in total. For each n, the values of $\overline{X}_{n,i}$ and $\hat{\sigma}_{\overline{X}_{n,i}}$, with $i = 1, \ldots, B$, can be computed based on these samples. From these values the values $P_{n,1}, \ldots, P_{n,B}$ are computed, so for each of the B samples of size n the probability that μ lies in the interval of the NDW of that sample is known. Figure 27(a) shows the probabilities $P_{n,i}$ for each percentage of vehicles equipped with a uCAN, together with the average probabilities $\overline{P}_n$, represented by the red line.



(a) 1000 values of $P_{n,i}$ for each $\frac{n}{N}$ (b) Average probabilities $\overline{P}_n$ for each $\frac{n}{N}$

Figure 27: Probabilities that μ is contained in the interval of NDW

As mentioned above, the necessary minimum percentage of vehicles with a uCAN equals the minimum percentage $\frac{n}{N}$ such that $\overline{P}_n \geq 0.95$. As can be seen from Figure 27(b), for the considered minute this percentage is equal to 21.9%. This means that when at least 12 vehicles were equipped with a uCAN, the estimator of traffic speed is sufficiently accurate.

Compared to the necessary minimum percentage for this specific minute found in Section 6.2.1, the percentage obtained in this section is smaller. Since the upper bound (26) used in the previous section is indeed stricter than the accuracy requirement of NDW, the percentage of estimators that will be contained in the interval of NDW is bigger than the percentage of estimators that satisfies (26), and will therefore be bigger than the necessary 95%. The two answers can be compared by drawing 50000 times 11 (20.8%), 12 (22.6%) and 13 (24.5%) vehicles from the total number of 53 vehicles of the considered minute. For every draw the mean of the speeds of these 11, 12 and 13 vehicles are compared to the true mean. In the table below, the percentages of estimators that satisfied the accuracy requirement of NDW are listed.

n	11	12	13
Percentage	94.8%	96.2%	97.3%

Table 5: Percentage of $\overline{X}_n \in [0.95\mu, 1.05\mu]$

As can be seen in Table 5, the percentage of estimators of the traffic speed that deviate 5% from μ is higher than the 95% requirement of NDW when 13 vehicles are equipped with a uCAN. For this minute, the minimum number of vehicles that have to be equipped with a uCAN to accurately estimate the traffic speed is less than 13. When 12 vehicles are equipped with a uCAN, 96.2% of the estimators are close enough to the true speed mean. Although this percentage seems relatively high, it should be noted that the actual necessary minimum percentage of this section is 21.9%, which corresponds to 11.6 vehicles. This number is rounded up to 12 vehicles, so a bigger percentage than necessary is used, which explains the higher percentage in the table. When 11 vehicles were equipped with a uCAN, the NDW requirement is not satisfied, shown by the percentage lower than 95% in the table. It can therefore be concluded that, for this minute, indeed 12 vehicles have to be equipped with a uCAN to accurately estimate the traffic speed.

Similar to the previous section, this is only a result for the specific minute that was considered. The obtained percentage is not valid for other minutes, where the value of c_v might be different. In the next section, the method described in this section will be applied to different minutes, with different values of N and/or c_v. For each of these minutes the necessary minimum percentage of vehicles is computed. Based on these results, a conclusion will be drawn for the necessary minimum percentage when N and c_v are not known.

6.2.3 Total number of vehicles not known

In the previous sections it is assumed that N is known to determine the necessary minimum percentage of vehicles equipped with a uCAN based on real-time information. However, as mentioned in Section 6.1, when only uCAN data is used to estimate the traffic speed, the value of N is unknown, because only the vehicles with a uCAN are detected. In this section, where it is assumed that N is not known, the following question will be answered:

What is the minimum percentage of vehicles equipped with a uCAN that is necessary to accurately estimate the traffic speed when the analysis is based on real-time information obtained from uCAN data?

To answer this question, the method from Section 6.2.2 will be applied to each minute of 6 days, i.e. 8640 minutes. For each minute the necessary minimum percentage of vehicles with a uCAN is computed. In Figure 28 this minimum percentage is plotted against the total number of vehicles N of the corresponding minute.

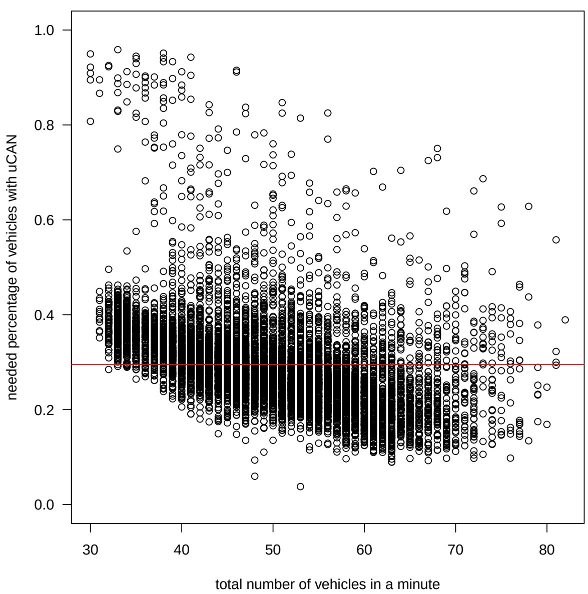


Figure 28: Necessary minimum percentage of vehicles with a uCAN for different minutes

As can bee seen in the figure, for some minutes the necessary minimum percentage is relatively high. The reason for this can be found in the speeds of these minutes. In these minutes, the differences between the speeds were relatively big, which was a result of the beginning (or ending) of a congestion, leading to high speeds at the start (end) and low speeds at the end (start) of the minute. The standard deviation of these minutes are therefore relatively large, so a relatively high percentage of vehicles with a uCAN is necessary to accurately estimate the traffic speed.

From these minimum percentages, the answer to the question of this section can be obtained. First, consider a fixed number N. For the different minutes with this total number of vehicles, different minimum percentages of vehicles with a uCAN are necessary. Similar to Section 6.2.2, where the true probability that μ lies in the interval is estimated by the average value of the observed probabilities, the true necessary minimum percentage of vehicles with a uCAN in a minute with a total number of j vehicles,, denoted by q_j with $j = 30, \ldots, 82$, can be estimated by the average value of the observed minimum percentages $\overline{Q}_j$ in the minutes with j vehicles in total. Since the total number of vehicles is assumed to be unknown, this result will be extended to an estimate of the true necessary minimum percentage of vehicles with a uCAN when the total number is not known.

Let q be the true necessary minimum percentage of vehicles with a uCAN, which can be written as

$$q = \sum_{j=30}^{82} q_j P(N = j).$$

Furthermore, let $\overline{Q}$ be the average value of all observed necessary minimum percentages. Similar to before, it can be shown that $\overline{Q}$ is an unbiased estimator for q, since

$$E[\overline{Q}] = \sum_{j=30}^{82} E[\overline{Q}|N = j]P(N = j) = \sum_{j=30}^{82} E[\overline{Q}_j]P(N = j) = \sum_{j=30}^{82} q_j P(N = j) = q,$$

where it is used that $\overline{Q}_j$ is an unbiased estimator for q_j. The necessary minimum percentage is therefore estimated by the average value of the minimum percentages shown in Figure 28, which is equal to 29.5%, shown by the red line in the figure. The answer to the question of this section can then be given:

The minimum percentage of vehicles equipped with a uCAN from the total number of vehicles in an arbitrary minute that is necessary to accurately estimate the traffic speed, where the analysis is based on real-time information obtained from uCAN data, is equal to 29.5%*.*

It can be seen from Figure 28, that much more points are above the red line for smaller values of N than for larger values of N, which was also noted in Section 6.1.2 and shown in Table 3. For this reason, also the average value of the necessary minimum number of vehicles of the minutes in the figure are considered. This average value is equal to 14.3, and shown as the red line in Figure 29 together with the necessary minimum numbers for each minutes of the four days.

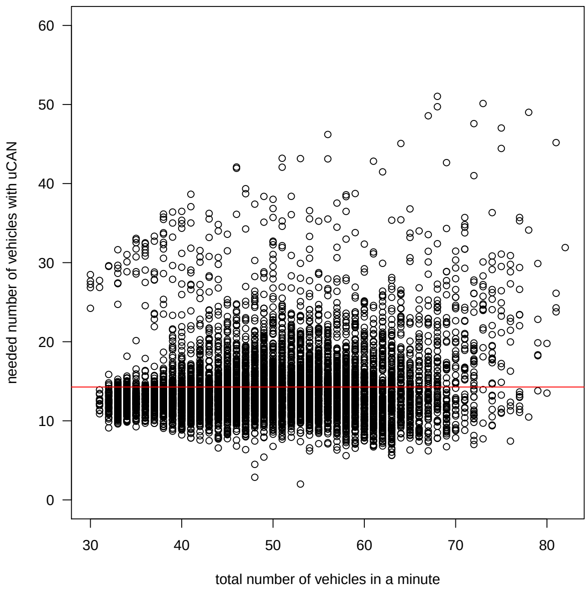


Figure 29: Necessary minimum number of vehicles with a uCAN for different minutes

The figure shows that when the necessary minimum numbers are considered, the amount of points above the average value does not differ much for different values of N. This result is therefore more general, since in this case the actual value of N has less influence. However, an absolute answer will in practice not be useful, since it can then not easily be determined how many vehicles in the Netherlands need to be equipped with a uCAN. In contrast, when the answer is given as a necessary minimum percentage, this percentage can be translated to a necessary minimum percentage of all vehicles in the Netherlands, which is shown in Chapter 7. For this reason, the necessary minimum number will no longer be considered.

Compared to the answer of Section 6.1.2, the obtained necessary minimum percentage in this section is smaller. These percentages are compared in a simulation study, where for both percentages 1440 minutes are simulated. Based on a random draw of the respective percentages of vehicles with a uCAN from the total number of vehicles in each minute, the estimator for the traffic speed is computed. Since also the real traffic speed is known in the simulation, the percentage of minutes for which the estimator is contained in the corresponding interval of NDW can be determined. These percentages are equal to 95.3% and 96.1% for the necessary minimum percentage of this section and the necessary minimum percentage of Section 6.1.2 respectively. Since the former percentage still results in an accurate estimator for the traffic speed, the lower percentage obtained in this section can be used. Moreover, in this section real-time information about the vehicles, i.e. the uCAN data, is used, which is of interest in this thesis. For these reasons, the necessary minimum percentage obtained in this section will be used in the next chapter to check if this percentage also holds in practical settings.

7 The necessary minimum percentage applied to practical settings

The result of Section 6.2.3 will be applied to practical settings in Section 7.1, to see if the obtained necessary minimum percentage of vehicles with a uCAN also holds in these settings. This minimum percentage is based on the accuracy requirement of NDW. However, research done in [36] shows that the traffic speed estimated with the data from inductive loop detectors does not meet this requirement. In that research, data from laser radars at 75 locations in the Netherlands are used to assess the accuracy of the data from inductive loop detectors. The results showed that in 11% of the minutes the traffic speed obtained from data from detectors has a relative uncertainty bigger than 8%. This means that, nowadays, the estimated traffic speeds do not satisfy the NDW requirement. As a result, even with a lower percentage of vehicles equipped with a uCAN, the traffic speed estimated with these vehicles will still be more accurate than the traffic speed estimated with data from detectors. Since then less vehicles need to be equipped with a uCAN to meet the accuracy of inductive loop detectors, this would mean that uCAN data can be used for the estimation of the traffic speed in an earlier stage of the installation of the uCAN into vehicles. Therefore, it is investigated in Section 7.2 what the necessary minimum percentage of vehicles with a uCAN is when a weakened requirement for the accuracy of the estimator is used.

7.1 Practical settings

In Section 6.2.3, where the analysis is based on real-time information, it is concluded that the traffic speed is accurately estimated with the mean of the speeds of the vehicles with a uCAN if in each minute 29.5% of the vehicles is equipped with a uCAN. Since the vehicles with a uCAN will not be equally divided over the Dutch road network, it is not likely that in each minute the same percentage of the vehicles is equipped with a uCAN. To check if the result of the previous chapter holds in a practical setting, for each minute the binomial distribution is used to randomly draw the number of vehicles with a uCAN in that minute, independent of the number of vehicles with a uCAN in the other minutes. This setting will be referred to as Setting 1. Since the expected value of a binomial distribution is the product of its parameters, it is chosen that

$$n \sim \text{Binom}\left(N, 0.295\right),$$

because then $\text{E}[n] = 0.295N$. The percentage of vehicles that is equipped with a uCAN in one minute is therefore on average equal to 29.5%. Two days are simulated in Fosim and for each minute the number n of vehicles equipped with a uCAN is determined by drawing a realisation from the above-mentioned binomial distribution, with N the total number of vehicles in the specific minute. After that, the vehicles with a uCAN are determined by randomly drawing n vehicles from the N vehicles in that minute. The traffic speed is then estimated with the mean $\overline{X}_n$ of the speeds of these n vehicles. Since the data is simulated, all data of the considered days is known. As a result, the speeds of the N vehicles in every minute are known, and thus the real traffic speed μ can be calculated for every minute. For each of the minutes, it can therefore be checked if the estimated traffic speed $\overline{X}_n$ is contained in the NDW interval $[0.95\mu, 1.05\mu]$. Figure 30 shows the estimated traffic speed and its corresponding interval for the first 25 minutes.

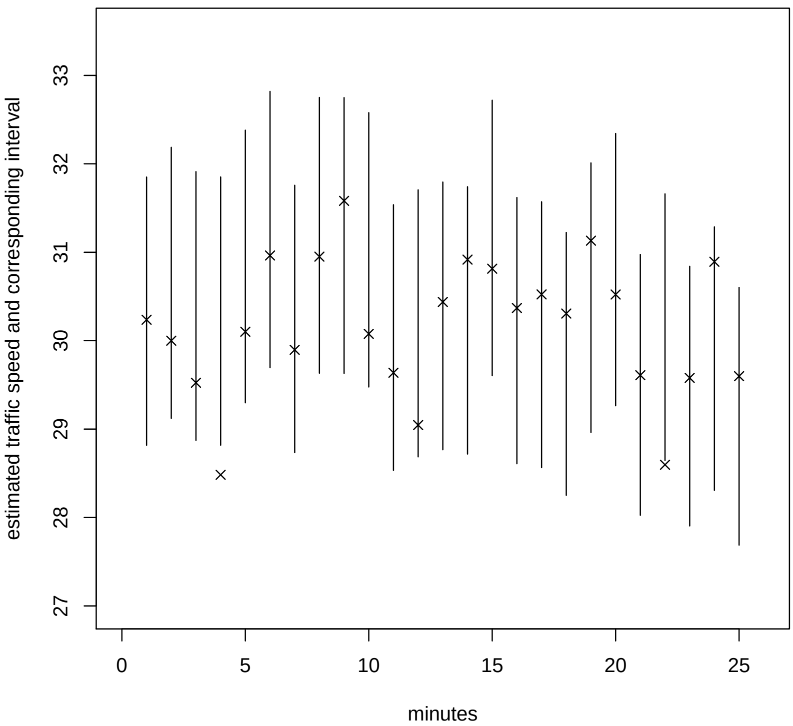


Figure 30: Needed vehicles with a uCAN for different minutes

The figure above shows that for example for the fourth minute, the estimator is not contained in the corresponding interval. However, for most of the first 25 minutes the interval contains the estimated traffic speed. When it is checked for each minute of the considered days whether the estimator of the traffic speed lies in its corresponding interval, the percentage of minutes in which the estimator is contained in the interval can be calculated. The resulting percentage is equal to 95.4%. This means that if on average 29.5% of the vehicles in one minute at a specific point is equipped with a uCAN, in 95.4% of the minutes the traffic speed is accurately estimated. The NDW requirement that

$$P\left(0.95\mu \leq \overline{X}_n \leq 1.05\mu\right) \geq 0.95$$

is therefore satisfied. However, this percentage is obtained by randomly drawing a sample of size n, which is also random, from the total number of vehicles for each minute. By repeating these simulations, different percentages will therefore be obtained. When this simulation is repeated 100 times, the average percentage is equal to 95.2%, with a standard deviation of 0.3%. Also this average percentage indicates that the requirement of NDW is satisfied.

For the result above, the number of vehicles with a uCAN is for each minute randomly drawn from a binomial distribution, such that the average percentage of vehicles with a uCAN in one minute is equal to 29.5%. Moreover, this number for a specific minute is drawn independent of the number of vehicles with a uCAN in the other minutes. However, in practice, the numbers of vehicles with a uCAN of different minutes are not independent, since the total number of vehicles that is equipped with a uCAN is fixed. Furthermore, it is possible that relatively many vehicles that are equipped with a uCAN belong to business persons for example. Although these people are driving relatively often on the highway, the moment that these vehicles are on the highway is not equally divided over the day, since relatively many of these vehicles will be driving on the highway during rush hour. Due to this and other reasons, the differences in percentages of vehicles equipped with a uCAN may differ much during the day. For the reasons mentioned above, the result of the previous chapter is also checked for a more practical setting, which will be referred to as Setting 2. More specific, this result is checked when in advance 29.5% of all vehicles that pass a specific point in the 48 hours are randomly equipped with a uCAN. This means that it is fixed in advance which vehicles are equipped with a uCAN, and thus also how many vehicles are equipped. With this setting, there will be minutes with relatively many vehicles with a uCAN, which represent minutes in rush hour, and minutes with a relatively small percentage of vehicles with a uCAN. The traffic speed in a specific minute is then estimated with the speeds of the vehicles in the considered minute that were in advance equipped with a uCAN. Since again the real traffic speed can be computed, the percentage of estimated traffic speeds that are contained in the corresponding interval of NDW can be computed. For the two considered days, this percentage is equal to 94.8%. Since again this percentage is different for different samples, the simulations are repeated 100 times to obtain an average percentage. For this setting this average percentage is equal to 95.1%, with a standard deviation 0.4%. Note that this percentage is close to the average percentage of Setting 1, which can be explained by the fact that both settings are similar, with the main difference that in the second setting the total number of vehicles with a uCAN is fixed. A result of this difference is that in the second setting a relatively low percentage of vehicles with a uCAN in one minute will be compensated by a relatively high percentage of vehicles with a uCAN in another minute. In the previous setting, though, it is possible that more than, or less than, 29.5% of the total number of vehicles in the 48 hours is equipped with a uCAN. Although the difference of the two settings in the resulting percentage is small, the standard deviation of the observed percentages is in this setting slightly bigger than the standard deviation of the previous setting. However, since the average percentage is bigger than 95%, the NDW requirement is also satisfied in this setting.

The idea of this last setting, with a fixed total number of vehicles equipped with a uCAN, can be extended to all vehicles in the Netherlands. On 1 January 2013, 8959028 vehicles were registered in the Netherlands, see [35]. In the two settings above, all vehicles of the two considered simulated days have passed the specific point where the traffic speed is estimated. However, in practice not all vehicles pass the same point of a highway. Some vehicles will only be driving in the north of the Netherlands for example, and will therefore not pass a specific point on a highway in the south. Also vehicles that are driving in the area near the considered highway will not necessarily drive on the highway, since these vehicles could be driving in the centre of a city, parked in a residential garage or repaired by auto mechanics for example. For this reason, the result of the previous chapter is checked for this generalization to all vehicles in the Netherlands as well. This means that in advance, 29.5% of the total number of vehicles in the Netherlands is equipped with a uCAN. In this setting, referred to as Setting 3, not all vehicles that are equipped with a uCAN will actually pass the specific point. It is therefore possible that relatively few, or relatively many, vehicles of the total number of vehicles in the simulation are equipped with a uCAN. The resulting percentage of minutes for which the traffic speed is accurately estimated with the mean of the speeds of the vehicles with a uCAN in that minute will therefore vary relatively much compared to the previous two settings. When this simulation is repeated 100 times, the average percentage is equal to 95.3% with a standard deviation of 0.6%. This higher standard deviation can be explained, since in this setting not only the percentage of vehicles with a uCAN in one minute can be relatively low or high, but also the percentage of vehicles with a uCAN in the two considered days can be relatively low or high.

As a summary, the resulting average and standard deviation of the percentages of minutes for which the traffic speed is accurately estimated are given in Table 6.

	Setting 1	Setting 2	Setting 3
Average percentage	95.2%	95.1%	95.3%
Standard deviation	0.3%	0.4%	0.6%

Table 6: Percentages of accurately estimated minutes

Since the three average percentages shown in the table above are all bigger than 95%, the necessary minimum percentage of vehicles with a uCAN satisfies the NDW requirement in the practical settings. The answer to the main question of this thesis can then be given:

The minimum percentage of vehicles equipped with a uCAN that is necessary to accurately estimate the traffic speed is equal to 29.5%, where this percentage can be interpreted in three ways: either as 29.5% of the vehicles in each minute, or as on average 29.5% of the vehicles in each minute or as 29.5% of all vehicles in the Netherlands needs to be equipped with a uCAN.

In the next section, an answer to the main question will be given, where a requirement weaker than the NDW requirement will be used.

7.2 Weakened requirement of NDW

The result obtained in the previous section implies that when, in the future, 29.5% of all vehicles in the Netherlands is equipped with a uCAN, uCAN data can be used to accurately estimate the traffic speed. This then means that in the future, from the moment that real-time data of 29.5% of the vehicles is known, inductive loop detectors are no longer necessary to obtain data for the estimation of traffic speeds. As mentioned above, this percentage will lead to an estimation of the traffic speed that is more accurate than the current estimation based on data from inductive loop detectors. As a consequence, the traffic speed estimated with uCAN data will still be more accurate than the estimation with detectors when a smaller minimum percentage of vehicles is equipped with a uCAN. Therefore, it is investigated in this section what the necessary minimum percentage of vehicles with a uCAN is when a weakened requirement for the accuracy of the estimator is used. To ensure that the traffic speed estimated with uCAN data is still more accurate than the traffic speed estimated with data from detectors, the weaker requirement is chosen to be:

$$P\left(0.95\mu \leq \overline{X}_n \leq 1.05\mu\right) \geq 0.90. \tag{29}$$

Although the same relative uncertainty of the NDW requirement is chosen, namely 5%, it is required that in only 90%, instead of 95%, of the minutes the estimator is contained in the corresponding interval. With this weakened requirement, the following question can be stated:

What is the minimum percentage of vehicles equipped with a uCAN that is necessary to accurately estimate the traffic speed when the analysis is based on real-time information obtained from uCAN data and the weakened requirement is used?

To answer this question, the method described in Section 6.2.3 will be used. Similar to that section, the minimum percentage of vehicles with a uCAN is determined for each minute, where the simulated data of the 6 days in the previous chapter is used. The necessary minimum percentage of vehicles with a uCAN is then equal to the average of these minimum percentages. Figure 31 shows the minimum percentage of vehicles with a uCAN for each minute plotted against the total number of vehicles in that minute, together with the average minimum percentage, shown by the horizontal red line.

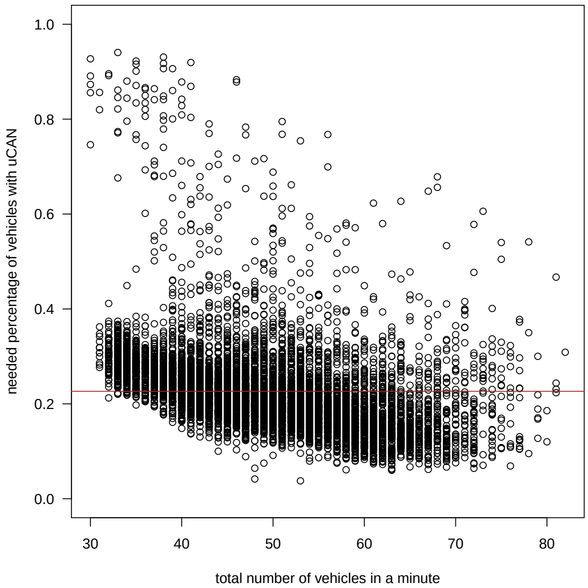


Figure 31: Necessary minimum percentage with the weaker requirement (29)

Compared to Figure 28, the minimum percentages, and thus the average value of these percentages, are lower, which is a result of the less strict requirement. The necessary minimum percentage of vehicles that need to be equipped with a uCAN to accurately estimate the traffic speed, is equal to the average value 22.6%. This percentage is also checked by repeating the simulation for the three settings 100 times as described in the previous section. The resulting average and standard deviation of the percentages of minutes for which the traffic speed is accurately estimated can be found in Table 7 below.

	Setting 1	Setting 2	Setting 3
Average percentage	90.5	90.1	90.4
Standard deviation	0.4%	0.6%	0.7%

Table 7: Percentages of accurately estimated minutes based on the weaker requirement (29)

In the table above, a result similar to the previous section can be observed. The three average percentages are all close to the required 90%, while the standard deviation slightly increases. It can therefore be concluded from the table, that the obtained necessary minimum percentage of vehicles with a uCAN also holds in the practical settings. The answer to the question of this section can then be given:

With the weakened requirement of NDW (29)*, the minimum percentage of vehicles equipped with a uCAN that is necessary to accurately estimate the traffic speed, where the analysis is based on real-time information obtained from uCAN data, is equal to* 22.6%*, where this percentage can again be interpreted in the same three ways as above.*

Since the inductive loop detectors do not satisfy the required relative uncertainty as well, it can also be investigated what the necessary minimum percentage of vehicles with a uCAN is when the relative uncertainty is allowed to be bigger. The NDW requirement is then weakened to the requirement that

$$P\left(0.90\mu \leq \overline{X}_n \leq 1.10\mu\right) \geq 0.95. \tag{30}$$

In other words, it is required that in at least 95% of the minutes the estimator of the traffic speed differs at most 10%, instead of 5%, from the real traffic speed. The minimum percentages of vehicles with a uCAN with this requirement, again obtained by applying the method of Section 6.2.3, can be found in Figure 32.

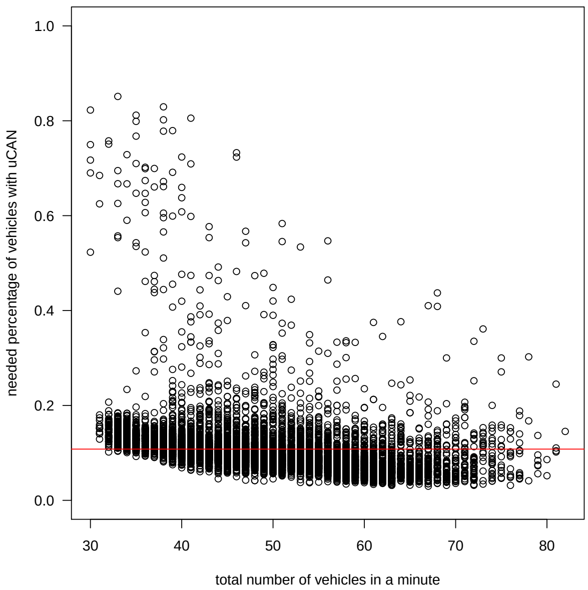


Figure 32: Necessary minimum percentage with the weaker requirement (30)

As can be clearly seen from the figure, the obtained minimum percentages are much smaller than the minimum percentages obtained with the NDW requirement or the previous weakened requirement. The average value of the minimum percentages of vehicles with a uCAN, shown by the horizontal red line, is equal to 10.8%. This percentage is about half the value of the percentage obtained with the previous weakened requirement and even about three times smaller than the percentage obtained with the NDW requirement. A relatively small change in the requirement can thus result in a relatively big change in the resulting necessary minimum percentage. The result for the second weakened requirement is again checked for the three practical settings, resulting in the following table:

	Setting 1	Setting 2	Setting 3
Average percentage	95.3	95.1	95.2
Standard deviation	0.3%	0.5%	0.6%

Table 8: Percentages of accurately estimated minutes based on the weaker requirement (30)

Also the result for this weakened requirement of NDW, the obtained necessary minimum percentage holds in the practical settings. The answer to the question of this section, where in this case the weakened requirement (30) is used, can then be given:

With the weakened requirement of NDW (30)*, the minimum percentage of vehicles equipped with a uCAN that is necessary to accurately estimate the traffic speed, where the analysis is based on real-time information obtained from uCAN data, is equal to* 10.8%*, where this percentage can again be interpreted in the same three ways as above.*

The answers given in this section are all answers to the main question of this thesis, where different requirements for the accuracy of the estimator are used. The results of this, and the previous, chapter can be summarized in Table 9 below. In this table, the requirement that is used is expressed in terms of the estimator of the traffic speed $\overline{X}_n$ and the real traffic mean μ. On the right column of the table, the necessary minimum percentage of vehicles with a uCAN can be found.

Requirement	Necessary minimum percentage
$P\left(0.95\mu \leq \overline{X}_n \leq 1.05\mu\right) \geq 0.95$	29.5%
$P\left(0.95\mu \leq \overline{X}_n \leq 1.05\mu\right) \geq 0.90$	22.6%
$P\left(0.90\mu \leq \overline{X}_n \leq 1.10\mu\right) \geq 0.95$	10.8%

Table 9: Percentages of accurately estimated minutes based on the weaker requirement (30)

The necessary minimum percentages of vehicles with a uCAN apply to all three practical settings that are discussed in this chapter. This means that when the percentage of vehicles, either in each minute or in the Netherlands, that is equipped with a uCAN is equal to the mentioned percentages in Table 9, the traffic speed will be accurately estimated based on the uCAN data, for the corresponding accuracy requirement.

8 Conclusion

Traffic management plays an important role in reducing congestion on highways and improving the traffic flow. For traffic management, applications such as adjusting the speed limit or open extra lanes at rush hour are used to reduce congestion. These applications make use of real-time data to analyse and control the current traffic situation. Nowadays, real-time information is mostly obtained from inductive loop detectors. However, data that is stored in the CAN bus of vehicles contains much more information, such as steering position, speed and directional indicators. This detailed in-vehicle data can be obtained by equipping vehicles with a uCAN, a universal module placed inside a vehicle, and can thus also be used as real-time data. To contribute towards the process of using in-vehicle data for traffic management, the use of uCAN data to estimate the traffic speed is investigated.

However, since the uCAN is a new module, vehicles are not equipped with a uCAN yet and thus uCAN data is not available. To analyse the estimation of the traffic speed with vehicles equipped with a uCAN, the uCAN data is therefore emulated. To obtain data that will be used for the emulation of uCAN data, the simulation model Fosim is used, which is calibrated and validated for Dutch highways. With this model, traffic is simulated on a road section consisting of a two-lane highway and one entrance ramp. In this simulation, the traffic intensity is chosen such that at least 30 vehicles are passing a certain point in one minute. The positions and speeds of each vehicle at corresponding times is obtained in Fosim by placing inductive loop detectors, resulting in data at fixed locations. Since uCAN data of a vehicle is given each second, this Fosim data is processed in order to emulate the relevant uCAN data that is used for the estimation of the traffic speed. This is done by interpolation of the position and speed at given time knots. For the interpolation of the given speeds, the natural cubic spline is used, and for the interpolation of the given position, the cubic Hermite spline is used. This last interpolant is chosen since then also the known derivatives, i.e. the speeds, can be used. These interpolants are discretized such that the position and speeds are known per second. After this discretization, the uCAN data is emulated and used for the estimation of the traffic speed.

The emulated uCAN data, which is known for each second, is interpolated such that the position and speed of a vehicle is known at every arbitrary time between the first and last moment that the vehicle drives on the highway. For each vehicle, the time that the vehicle passes the considered point is determined with the interpolant of the positions, after which the speed of the vehicle at that point is determined with the interpolant of the speeds. When this is done for each vehicle, the uCAN speeds are obtained. However, it is assumed that only n of the total number of vehicles that pass the point in one minute are equipped with a uCAN. The traffic speed in that minute is then estimated with the average speed of the uCAN speeds of these n vehicles. In this thesis it is assumed that this estimator is normally distributed, which appeared to be reasonable. To determine the necessary minimum percentage of vehicles equipped with a uCAN for an accurate estimation of the traffic speed, the accuracy requirement of NDW is used, which requires that in at least 95% of the minutes the estimator of the traffic speed differs at most 5% of the real traffic speed. The necessary minimum percentage of vehicles with a uCAN is then determined by the minimum percentage for which the estimator satisfies this requirement.

Whether the estimator satisfies the NDW requirement depends on the coefficient of variation of the estimator. Since this standard deviation is not known, two methods are developed to obtain information about the value of the standard deviation. For the first method, where the coefficient of variation is considered as a random draw from the historical values, the necessary minimum percentage is first determined for each minute. On the basis of this percentage, the conclusion is made that the minimum percentage of vehicles equipped with a uCAN that is necessary to accurately estimate the traffic speed of an arbitrary minute is equal to 30.6%. The analysis of the second method is based on real-time information, which is obtained from the vehicles with a uCAN that pass the considered point where the traffic speed is estimated. After first determining the necessary minimum percentage for each minute, it is concluded that the minimum percentage of vehicles equipped with a uCAN that is necessary to accurately estimate the traffic speed of an arbitrary minute is equal to 29.5%. As can be seen from both results, the necessary minimum percentage of vehicles with a uCAN is lower when the second method is used. From a simulation study it followed that the traffic speed is also accurately estimated when the percentage of vehicles that is equipped with a uCAN is equal to the necessary minimum percentage obtained with the second method, which is a lower percentage than the percentage obtained with the first method. The method based on real-time information about each minute therefore gives a more accurate result.

In both methods it is assumed that in each minute the resulted percentages of vehicles are equipped with a uCAN. This result is checked for a practical setting, where 29.5% of the vehicles in the Netherlands is equipped with a uCAN. This means that not all vehicles are passing the point where the traffic speed needs to be estimated. For this generalization, it is also concluded that when 29.5% of all vehicles in the Netherlands is equipped with a uCAN, the traffic speed is accurately estimated.

All results are based on the accuracy requirement of NDW. Since the traffic speed computed with data from inductive loop detectors does not satisfy this requirement, the accuracy requirement of NDW is weakened. With this weaker requirement, a lower percentage of vehicles with a uCAN is obtained, which means that uCAN data can be used in an earlier stage to estimate the traffic speed. The first weakened requirement is that in at least 90% of the minutes the estimator of the traffic speed differs at most 5% of the real traffic speed. With this requirement, the necessary minimum percentage of vehicles equipped with a uCAN is equal to 22.6%, which means that when 22.6% of all vehicles in the Netherlands is equipped with a uCAN, the traffic speed will be accurately estimated. An alternative weakened requirement is that in at least 95% of the minutes the estimator of the traffic speed differs at most 10% of the real traffic speed. The necessary minimum percentage of vehicles in the Netherlands that need to be equipped with a uCAN, for which the estimator satisfies this requirement, is equal to 10.8%. As can be seen from both resulting percentages, the necessary minimum percentage of vehicles with a uCAN decreases significantly when either the probability or the relative uncertainty of the NDW requirement is increased.

It can be concluded from the results of this thesis that if in the future 29.5% of the vehicles in the Netherlands is equipped with a uCAN, the estimator for the traffic speed based on the uCAN data from these vehicles satisfies the requirement of NDW. This means that the estimated traffic speed differs at most 5% from the real traffic speed in at least 95% of the minutes. When in the future the mentioned percentage of all vehicles is equipped with a uCAN, the data from inductive loop detectors that is currently used for the estimation of traffic speeds, can be replaced by the data from the vehicles that are equipped with a uCAN. The uCAN data can then be used for traffic management applications related to traffic speed such that traffic management can be improved. Moreover, the estimator for the traffic speed based on this uCAN data will be more accurate than the estimator for the traffic speed based on the data from the detectors. For a requirement weaker than the NDW requirement, the necessary minimum percentage of vehicles with a uCAN is equal to 22.6% for requirement (29) and equal to 10.8% for requirement (30). So for a lower accuracy that is still higher than the accuracy of the estimator for the traffic speed that is used nowadays, even less vehicles need to be equipped with a uCAN, what means that then uCAN data can be used in an earlier stage.

9 Recommendations

In this chapter, recommendations for future research will be discussed.

- Since nowadays uCAN data is not available, this data is emulated in Fosim with the simulation settings described in Section 3.3. In this setting a two-lane highway is considered. Since the Dutch highways not only consist of two-lane highways, a recommendation is to consider a setting where a three-lane highway is chosen for example. Also, situations where vehicles suddenly has to reduce their speeds due to a situation, such as an accident, where a two-lane highway is changed in a one-lane highway. Furthermore, besides an entrance ramp, also an exit ramp could be considered. These settings will lead to more accurate emulated uCAN data, since then more practical situations are taken into account.
- In the simulation model Fosim, the maximum speed limit is 120 km/h. However, since 1 September 2013 the maximum speed limit for all highways in the Netherlands is changed to 130 km/h. Since the traffic speed depends on the maximum speed limit, it is recommended to update Fosim so that more accurate uCAN data can be emulated.
- To estimate the traffic speed at a specific point, the emulated uCAN speeds need to be interpolated, since these are given per second. In this thesis, for the interpolation of the speed knots obtained from Fosim, the natural cubic spline is used, since Fosim does not give accelerations of vehicles at given locations. Therefore, for the interpolation of the emulated uCAN speeds, the natural cubic spline is used as well. However, uCAN data contains the acceleration of vehicles and thus can be used in practice. When these accelerations are used, the speeds obtained from Fosim and the emulated uCAN speeds can be interpolated with the cubic Hermite spline. This will give a more accurate speed of the vehicle that passes the specific position. Furthermore, when in the future uCAN data is used for the estimation of the traffic speed, the accelerations of vehicles can be used to obtain a more accurate estimator for the traffic speed.
- In the previous recommendation, the acceleration that can be obtained from the uCAN is used to improve the estimations. Moreover, uCAN data contains much more detailed in-vehicle data that is not used in this thesis for the estimation of the traffic speed. Information such as the following distance could lead to a smaller percentage of vehicles that need to be equipped with a uCAN for an accurate estimation of the traffic speed. When less vehicles are needed for an accurate estimation, uCAN data can be used in an earlier stage without the weakening of the accuracy requirement of NDW. In the case that more uCAN data is used and that the weakening requirement is used as well, uCAN data can be used even earlier.
- For further research of uCAN data, it is recommended that other use cases of uCAN data are investigated that can be used to improve traffic management. For example, if uCAN data can be used for the improvement of traffic management applications for weather conditions on the road. Promising results can lead to the use of uCAN data in the future.

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